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Fine structure of the production in low to medium Reynolds number wall turbulence

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ABSTRACT

Palm statistics (so called after the Swedish mathematician Conny Palm (1907–1951)) are the statistics of a random quantity conditioned by another. Such statistics have already been used in the turbulence community, but not always adequately. Special care is indeed needed to correctly define these statistics in order to avoid biasing effects. The first part of this paper is devoted to the analysis of these peculiarities. Different characterizations of the Palm statistics, at directional and contour crossings are introduced afterwords. It is found that the level-crossing frequency and likewise the turbulence activity in the spanwise direction is larger than in the streamwise direction, and that the wall normal vorticity is more active than the streamwise one. The statistics of the local production conditioned either by the level crossings of the streamwise u or the wall normal local velocity v in a fully developed channel flow are analyzed by using direct numerical simulations data performed in large computational boxes similar to Hoyas&Jiménez (Phy. Fluids, 011,702, 2006). The Karman number (based on the shear velocity $\bar{u}_\tau$, channel half width h and kinematic viscosity ν) ranges from $Re_\tau = 180$ to $Re_\tau = 1100$. The aim here is first to determine the appropriate scaling of the conditional quantities that directly enter in the Palm production ensemble averages, and secondly, to analyze the effects of large scale and very large scale motions (LSM, VLSM) on these statistics. It is well known that VLSM transport a considerable amount of the shear stress, but the direct dynamic role of the latter in the production process remains still unclear. The structures that maintain coherence in the Palm statistics, in a universal Reynolds number independent way, scale with inner variables ν and $\bar{u}_\tau$ in the layer bounded by$y < 50\nu/\bar{u}_\tau$ from the wall. They are as long as $\Lambda_x = 5000\nu/\bar{u}_\tau$ for the v-level crossings and $\Lambda_x = 2000\nu/\bar{u}_\tau$ for the $u-$level crossings. Further away, in the $50\nu/\bar{u}_\tau < y < 200\nu/\bar{u}_\tau$ sublayer, the coherence streamwise lengths scale with outer variables as $3h$, a value which is typically the upper bound of the streamwise extend of vortex packets.

1. Introduction

The crucial role played by intermittent streamwise u and wall normal v velocity components in the production process near the wall is well known since the discovery of coherent structures in the early 1960′s (Kline et al., [1], and reported as early as the 1970 s by Corino and Brodkey [2], Willmarth and Lu [3] and Lu and Willmarth [4]. Intense events associated with the buffer layer penetrate deep into the outer flow (ejections) or into the viscous sublayers (sweeps) and induce high wall shear stress (Orlandi and Jiménez [5]). Most of the production of turbulent energy occurs during these intense events. The latter are best characterized by splitting $u-v$ into the quadrants [3], but a simpler way to detect ejections and sweeps is to consider events when u passes through a threshold, which is roughly equal to its local rms value (Bogard and Tiederman [6]). The first question that arises is how much an event characterized by a fixed value of u contributes to the production. The resulting conditional expected values as a function of the level of u allow, along with other information, a rigorous check of some models relating to the joint probability density function of u and v such as the joint Gaussianity. We will show in this paper that these statistics also allow a detailed investigation of the Reynolds number dependence induced by outer eddies, in a more refined way than provided by other methods such as the spectral analysis.

Consider a stochastic stationary signal $u(t)$ with zero mean and standard variation $\sigma_u = \sqrt{\overline{uu}}$, a constant ℓ_u, the derivative $u'(t) = du/dt$, the level crossings $u(t) = \ell_u\sigma_u$, and the number of level crossings $N_{\ell u}(T)$ in an interval of time T. This problem has a long story starting with Rice [7,8]. The level crossing frequency $f_{\ell_u} = N_{\ell_u}/T$ is given by the conditional expectation $f_{\ell_u} = p(u = \ell_u\sigma_u)E\{|u'|\,|u(t) = \ell_u\sigma_u\}$, where $p(u)$ is the probability density and E stands for the ensemble averaging. This expression applies to any signal providing that it is mean-zero separable stationary (Ylvisaker, [9]). The zero-crossings frequency of a Gaussian

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signal is noticeably simpler, with $f_{0_{u_G}} = \sqrt{\overline{u'^2}}/\pi\sigma_u$. The Liepmann scale associated with f_{0_u} is $Li = 1/\pi f_{0_u}$(Liepmann [10]) and it is equal to the Taylor scale for a Gaussian signal. Sreenivasan et al. [11] questioned this equivalence in wall- bounded turbulent flows wherein the Liepmann and Taylor scales were found to concur fairly well. There is also some similarity between the level crossing frequency of velocity fluctuations in the inner layer and the distribution $f_{\ell_{u_G}} = f_{0_u}\exp(-\ell_u^2/2)$ inferred from the Gaussian model (Tardu, [12]).

The cumulative probability distribution of time intervals $P(\tau)$ separating successive level crossings is a problem that remains unsolved even for Gaussian processes (Blake and Lindsey, [13]). First of all, the hypothesis that successive zero crossing intervals of Gaussian processes are independent is incorrect. In other words $P(\tau)$ deviates from Poissonianity for some normal signals. Kailasnath and Sreenivasan [14] noticed that $P(\tau)$ of zero crossings of streamwise velocity fluctuations u exhibits two exponentials from which a small and large time scale can be extracted. The large time scale is independent of the Reynolds number while the small one scales as $Re_\lambda^{-1/2}$ where Re_λ is the Reynolds number based on the Taylor scale λ. The wall normal velocity v and the Reynolds shear stress $-uv$ have just one time scale for reasons that are not well established. The dual time scales relevant to zero-crossings of u are also found in level crossings different from zero. This characteristic has so far been used to identify packets of vortical structures [15,16]. The $\ell_v \neq 0$ crossings of the wall normal v velocity fluctuations and the shear stress$-uv$ have only one time scale.

Palm distributions describe the statistics of a given quantity conditioned by level-crossings of another [17]. Quantities of interest are, amongst others, the probability density of a flow quantity q and its moments as a given component of velocity goes through an assigned threshold L. Care should be taken in applying these notions to turbulence. For instance, it has been argued that the zero-crossings of streamwise velocity should largely contribute to the isotropic part of dissipation in wall turbulence [14]. In this particular case the local flow quantity is $q = \varepsilon_{iso} = 15\nu(\frac{\partial u}{\partial x})^2$, where ν is the viscosity and x is the streamwise coordinate. Consider the $u(x, y, z; t)$ velocity component and to simplify, suppose that it is Gaussian. One can determine the zero-crossings of u along x for fixed timet, and coordinates y and z. The number of samples in an interval $[0, L_x]$ is $N_{0x} = L_x\sqrt{\overline{(\partial u/\partial x)^2}}/\pi\,\sigma_u$ in this case [7]. The mean dissipation $\overline{\varepsilon}_{iso}$ conditioned by $u = 0$ is obviously not the conventional expected E mean $\overline{\varepsilon}_{iso} = E(\varepsilon_{iso}|u = 0)$, because the number of samples N_{0x} is not statistically independent of ε_{iso}. The second example is the local production $q = P = -uv\frac{\partial\bar{U}}{\partial y}$ where v and $\bar{U}$ are respectively the fluctuating wall normal, and the mean velocity and y is the wall normal coordinate. For similar reasons, the mean production at u level crossings is a priori not $E(P) = -\ell_u\sigma_u\frac{\partial\bar{U}}{\partial y}E(v|u = \ell_u\sigma_u)$. Similar problems are of profound physical and theoretical interest. There is clearly an undeniable need to revisit this topic, because there is, in our opinion, a lack of precision and adequate formulation of level-crossing properties in turbulence related past research. Preliminary results on dissipation statistics at velocity level crossings have been reported in Tardu [18]. This paper is focused on the production characteristics in wall- bounded flows.

The paper is organized as follows. The Palm statistics are introduced in a simple way in 2.1. A detailed analysis of the level-crossings statistics is conducted in the Appendix through an original way. Different characterizations such as directional level crossings and contour crossings are introduced afterwords. Direct numerical simulations (DNS) performed in a turbulent channel flow in large computational domains are briefly described in Section 3. Some preliminary characteristics of the Palm statistics related to the local production are given in Sections 4.1 to 4.4. The latter are an overview of the results presented in detail in Tardu and Bauer [19] and are briefly summarized here with additional comments, to make the paper self-contained. The production characteristics conditioned by fixed amplitudes of the spanwise velocity fluctuations have recently been reported in Tardu [20]. The main topic of the present investigation is the scaling of the conditional local production statistics and the Reynolds number effects. Thus, the last sections of the paper deal with the effect and the repercussion of the large-scale passive structures on the ensemble averaged velocities conditioned by level crossings.

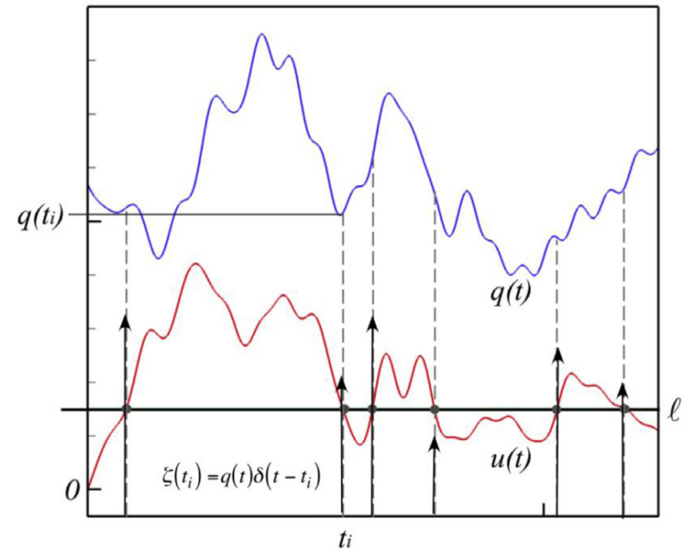

Fig. 1. Level crossings of the signal u marked by the random quantity q.

2. Level crossing statistics

2.1. One dimensional signals

Palm averages are conditional statistics. Consider two stochastic stationary signals $u(t)$ and $q(t)$. Fig. 1 schematically shows these processes. The aim is to determine the statistics of $q(t_i)$ conditioned by the events $u(t_i) = \ell_u\sigma_u$. One is at first tempted to determine the conditional mean $\overline{q}_{\ell ub}$ by computing simply

$$\overline{q}_{\ell_u b} = \frac{\sum_{i=1}^{N_{\ell u}} q(t_i)|u = \ell_u\sigma_u}{N_{\ell u}} = E\{q|u = \ell_u\sigma_u\} \tag{1}$$

i.e. by collecting all the $q(t_i)$ when the signal u crosses the level $\ell_u\sigma_u$, and performing the statistics over this specific data set. We recall that$E$ stands for the expected value and | refers to the conditional event. This procedure is correct if and only if the number of samples N_{ℓ_u} is statistically independent of the quantity q. However, as already indicated in the Section 1, N_{ℓ_u} in an interval [0, T] depends on the absolute derivative of $|u'| = |du/dt|$ and is given by:

$$N_{\ell_u} = T f_{\ell_u} = T\,p(u = \ell_u\sigma_u)E\left\{\left|u'\right|\;|u = \ell_u\sigma_u\right\} \tag{2}$$

Consequently, if $q(t)$ and $|u'|$ are correlated, then N_{ℓ_u} depends on the quantity that is of interest and the Eq. (1) leads to biased moments. Thus, the average $\overline{q}_{\ell_u}$ at level-crossing points is the normalized mean of q weighted by the absolute velocity derivative, i.e.

$$\overline{q}_{\ell_u} = \frac{E\{|u'|q|u = \ell_u\sigma_u\}}{E\{|u'||u = \ell_u\sigma_u\}} = \frac{\sum_{i=1}^{N_{\ell u}}|u'|(t_i)q(t_i)|u = \ell_u\sigma_u}{\sum_{i=1}^{N_{\ell u}}|u'|(t_i)|u = \ell_u\sigma_u} \tag{3}$$