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Behavioral crowds: Modeling and Monte Carlo simulations toward validation☆

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ABSTRACT

A mesoscopic model of behavioral crowds is developed within the framework of the kinetic theory for active particles. An analytic long-time equilibrium solution is obtained which gives a fundamental density-velocity diagram consistent with the empirical evidence. Numerical simulations based on a Monte Carlo particle method show that the proposed model has the capability to qualitatively depict emerging behaviors and to provide a realistic description of the crowd dynamics in complex evacuation scenarios.

1. Introduction

Crowd dynamics has received a growing attention in the last two decades not only for its theoretical interest but also for the potential societal benefits. A realistic modeling of the pedestrian's behavior can permit, for instance, to improve the design of buildings, aircraft and ships with respect to their safety in the event of an emergency evacuation and/or to understand how to optimize pedestrian flow in different situations.

It is well known that the modeling of crowd dynamics can be developed at three representation scales, namely microscopic, macroscopic, and mesoscopic [1]. A valuable reference concerning, modeling, mathematical problems, and applications related to the microscopic and macroscopic scales is provided by Cristiani et al. [2]. Microscopic models assume that the dynamics of the crowd emerges from the movement of individuals [3]. The state of the crowd is thus defined by specifying position and velocity of individuals and the dynamics is predicted on the basis of rules by which interactions between individuals are modeled. In contrast, the focus of the macroscopic models is on the crowd as a whole [4]. Accordingly, the state of the crowd is described with aggregate observables, such as density and velocity, and the dynamics is governed by balance equations for mass and/or momentum closed through phenomenological assumptions. Mesoscopic models are situated at an intermediate level between these two scales [5]. The state of the crowd is described by means of a probability distribution function over the microscopic state of individuals, namely position and velocity, while an additional internal variable is introduced to depict their heterogeneous behavioral strategy. Interactions between individuals are modeled at the micro-scale, while the aggregate observables are obtained by weighted moments of the aforesaid probability distribution.

A critical analysis of the advantages and drawbacks of the different scales selected for the modeling approach are discussed in [1], where it is concluded that the present state of the art does not yet allow well defined hallmarks to support an optimal choice. Microscopic models are shown to provide realistic results. However, keeping track of the individual state of each pedestrian leads to numerical simulations which are computationally demanding.

☆ This paper is dedicated to Tayfun Tezduyar on the occasion of his 60th birthday.

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Moreover, the results obtained from such approach may not be easily interpreted in the absence of a higher level model. Indeed it may be difficult, or nearly impossible, to use data from microscopic observations to infer the crowd behavior in a similar but different situation. Macroscopic models are appealing in that they allow to investigate complex dynamics which otherwise would be very difficult to deal with. An example is the pedestrian flows coupled with vehicular traffic networks studied in [6]. On the other hand, the heterogeneous behavior of pedestrians gets lost in the averaging process needed to their derivation and therefore macroscopic models totally disregard this important feature. Mesoscopic models have the potential of providing the crucial ingredients towards an accurate description of a crowd viewed as a living, hence complex, system but, in order to achieve such an objective, further developments are needed both from the modeling and computational standpoints.

In the present paper, a mesoscopic model is proposed based on two previous contributions to the modeling of crowd dynamics by the kinetic theory of active particles [7,8], where the former proposes a modeling approach in unbounded domains, while the latter takes into account interactions with walls. More specifically, the model proposed in [8], is revisited so as to simplify its mathematical formulation and, at the same time, better reproduce the empirical evidence.

The validation of crowd models against reality is a challenging topic that is poorly treated in the literature, with a few exceptions such as, for instance [9–11]. The empirical evidence is quite limited for developing a detailed comparison and, in addition, most of the data are available at the macroscopic scale, while the modeling process needs a detailed understanding of the microscopic dynamics. Therefore, a strategy should be elaborated to exploit the available data at the best of the panorama they offer. In the present study, we assess the ability of the proposed model to reproduce the density-velocity diagram in steady flow conditions and to depict some collective emerging behaviors which are observed by experiments, namely the self-organized behaviors which leads to the creation of lanes in streets and the increasing of evacuation time in stressful conditions.

The presentation is proposed as follows. In Section 2, the mathematical formulation of the crowd mesoscopic model is described. In Section 3, an analytic long-time equilibrium solution is obtained which is shown to provide a fundamental density-velocity diagram consistent with the empirical evidence. In Section 4, after a brief introduction of the Monte Carlo particle simulation method, the proposed model has proved to be capable of reproducing self-organized collective crowd behaviors which are empirically observed. In Section 5, conclusions and future research directions are presented.

2. Mesoscopic models of social crowd

Let us consider a crowd in a venue $\mathcal{S}$. The crowd can be subdivided into n different groups of persons which develop their own strategy, such as walking toward different targets. We refer to these groups as functional subsystems (FSs). The state of the overall system is described by the one-particle distribution functions $f_i = f_i(t, \boldsymbol{x}, \boldsymbol{v})$ with $i = 1, \dots, n$, which are such that $f_i(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{x}\, d\boldsymbol{v}$ denotes the number of pedestrians of the i-th FS whose state, at time t, is in the elementary volume of the space of the microscopic states $[\boldsymbol{x}, \boldsymbol{x} + d\boldsymbol{x}]$ and $[\boldsymbol{v}, \boldsymbol{v} + d\boldsymbol{v}]$ with $\boldsymbol{x} \in \mathcal{S}$ and $\boldsymbol{v} \in \mathcal{V} := \left\{\boldsymbol{v} : \|\boldsymbol{v}\| < \xi_{\text{LIM}}\right\}$, where ξ_{LIM} is the limit velocity that a fast pedestrian can reach in the free flow condition.

Macroscopic quantities can be computed by velocity weighted moments of the distribution function. As an example, density and the mean velocity are given by

$$\rho_i(t, \boldsymbol{x}) = \int_{D_v} f_i(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v}$$
$$\text{and} \quad \boldsymbol{\xi}_i(t, \boldsymbol{x}) = \frac{1}{\rho_i(t, \boldsymbol{x})} \int_{D_v} \boldsymbol{v}\, f_i(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v}. \tag{1}$$

Global expressions are obtained by weigthed sum over the index labeling the FSs, that is

$$\rho(t, \boldsymbol{x}) = \sum_{i=1}^{n} \rho_i(t, \boldsymbol{x})$$
$$\text{and} \quad \boldsymbol{\xi}(t, \boldsymbol{x}) = \frac{1}{\rho(t, \boldsymbol{x})} \sum_{i=1}^{n} \rho_i(t, \boldsymbol{x}) \boldsymbol{\xi}_i(t, \boldsymbol{x}). \tag{2}$$

The derivation of the mathematical structure used in the present work refers to the theory reviewed in [5] where pedestrians are viewed as active particles and interactions are modeled by theoretical tools of stochastic game theory. The balance in the elementary volume of the phase space between the inlet and outlet fluxes due to the movement of the pedestrians in the space and their mutual interactions gives

$$\begin{aligned}(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}})\, f_i(t, \boldsymbol{x}, \boldsymbol{v}) &= \eta_A \Big(\int_{\mathcal{V}} \mathcal{A}[\rho, \boldsymbol{\xi}](\boldsymbol{v}_* \to \boldsymbol{v}) f_i(t, \boldsymbol{x}, \boldsymbol{v}_*)\, d\boldsymbol{v}_* - f_i(t, \boldsymbol{x}, \boldsymbol{v}) \Big) \\ &\quad \eta_B(\boldsymbol{x}) \Big(\int_{\mathcal{V}} \mathcal{B}(\boldsymbol{v}_* \to \boldsymbol{v}) f_i(t, \boldsymbol{x}, \boldsymbol{v}_*)\, d\boldsymbol{v}_* - f_i(t, \boldsymbol{x}, \boldsymbol{v}) \Big),\end{aligned} \tag{3}$$

where η_A, η_B are the interaction rates between walkers and between walkers and walls, and $\mathcal{A}, \mathcal{B}$ are the transition probability densities which model the decision processes based on which pedestrians modify their velocity. The interaction rate η_A is assumed to be constant over the whole space domain $\mathcal{S}$ while the interaction rate η_B is supposed to be space dependent since it is expected that walkers interact with walls only if they are sufficiently close to them. The main features of the transition probability densities are summarized in Table 1 and are described in detail in the following two subsections. It is worth noticing that in Eq. (3), square brackets have been used to denote the functional dependence of the transition probability density $\mathcal{A}$ on the local density and mean velocity. Therefore, in spite of its linear appearance, the proposed crowd model is a set of strongly nonlinear integro-differential equations.

2.1. Modeling interactions between walkers

Interactions between walkers are assumed to modify their dynamics firstly by changing the direction of movement and, afterwards, by modifying the speed

$$\mathcal{A}[\rho, \boldsymbol{\xi}](\boldsymbol{v}_* \to \tilde{\boldsymbol{v}}) = \mathcal{A}_v[\rho, \boldsymbol{\xi}](v_* \to v) \mathcal{A}_\theta[\rho, \boldsymbol{\xi}](\theta_* \to \theta) \tag{4}$$

where the velocity has been decomposed in speed and direction $\boldsymbol{v} = \{v, \theta\}$.

Three types of stimuli are assumed to contribute to the modification of walking direction, namely, the desire to reach a defined target, the attraction toward the mean stream and the attempt to avoid overcrowded areas. These are represented by the three unit vectors $\boldsymbol{\nu}^{(t)}$, $\boldsymbol{\nu}^{(s)}$, and $\boldsymbol{\nu}^{(v)}$, respectively. It is expected that at high density, walkers try to drift apart from the more congested areas moving in the direction of $\boldsymbol{\nu}^{(v)}$. Conversely, at low density, walkers head for the target identified by $\boldsymbol{\nu}^{(t)}$ unless their level of anxiety is high in which case they tend to follow the mean stream as given