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On three-dimensional ALE finite element model for simulating deformed interstitial medium in the presence of a moving needle

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The effect of inserted needle on the subcutaneous interstitial flow is studied. Our goal is to describe the physical stress affecting cells during acupuncture needling. The convective Brinkman equations are considered to describe the flow through a fibrous medium. Three-dimensional simulations are carried out by employing an ALE finite element model. Numerical studies illustrate the acute physical stress developed by the implantation of a needle.

1. Introduction

In recent years, computational techniques have been widely used by researchers to investigate and simulate biological flow within three dimensional context. Applications include blood flow models, air flow models in the respiratory tract, interstitial flow models, and chemical mediators transport. Most of the structure and fluid interactions have been considered with simplified rigid wall or deformable wall models.

Methods to predict flows that account for moving domains or domain deformability using the finite element method are based on fixed mesh methods or moving mesh methods. On the one hand, fixed mesh methods include the immersed boundary formulation [1] which relies on the description of solid phase by adding a force vector to the governing equations. A similar approach, known as the fictitious domain formulation [2,3], is based on the use of Lagrange multipliers to enforce kinematic condition on the solid phase or alternatively based on a penalty method [3]. Both methods track solid phase with a characteristic function or a level set function. These methods are well adapted to moving bodies in the fluid or fluid-structure computation with interface of a highly geometric complexity. The latest method has been implemented with FreeFem++ [4]. On the other hand, moving mesh methods include the Lagrangian method, the moving finite element (MFE) method [5,6], the deformation map method [7], the Geometric Conservation Law (GCL) method [8], the space/time method [9–12], and the Arbitrarily Lagrangian–Eulerian (ALE) method [13–15] for the solution of fluid dynamic problems. Note that the space-time finite element method can also be implemented in FreeFem++ in 1D and 2D.

Significant progress has been made in recent years in solving fluid-structure interaction problems in deformable domains using the ALE method. The mathematically rigorous ALE framework has been well accepted to be applicable to simulate transport phenomena in time and allows some freedom in the description of mesh motion. A theoretical analysis of the ALE method can be found in [16,17]. However, ALE equations are computationally expensive when considering a large domain because of the necessity of continuously updating the geometry of the fluid and structural mesh. Interface tracking with time discretization also raises some implementation questions. The implementation of the ALE method can be done in FreeFem++ [18].

Study of biological flows plays a central role in acupuncture research. For a description of the underlying acupuncture

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mechanism, one can refer to [19–21]. Interstitial flow models take into account interstitial fluid, cell membrane interaction, and fiber interactions [22]. Mastocytes, among other cells, are able to respond to fluidic stimuli via mechanotransduction pathways leading to the degranulation and liberation of chemical mediators [23]. Degranulation mechanisms include interaction of the cell membrane with interstitial and cytosolic flow [24]. Ion transport in narrow ion channels is another challenging task to model. Indeed, degranulation of chemical mediators upon stimulation can be triggered by a rapid Ca^{2+} entry in the cytosol [25].

Modeling the three-dimensional interstitial flow in tissues is extremely challenging for a large number of reasons: a complex geometry of the tissue, an accurate constitutive description of the behavior of the tissue, and flow rheology are only few examples. Macroscopic models developed for incorporating complex microscopic structure are essential for applications [22,25–28]. In the context of acupuncture, the interstitial flow has been modeled by the Brinkman equations in two-dimensional fixed domain [27,28] and two-dimensional deformable domain [19].

In this paper, a porous medium formulation of the interstitial fluid is presented for modeling mastocyte-needle interaction in deformable connective tissues. This formulation is based on the conventional ALE characteristic/Galerkin finite element model for an unsteady flow thought a porous medium modeled by the incompressible Brinkman's equations in a three-dimensional moving domain. The motion of the needle in the fluid is taken into account. The main features of the model can be summarized as follows:

1. The loose connective tissue of the hypodermis is constituted of scattered cells immersed in extracellular matrix. The extracellular matrix contains relatively sparse fibers and abundant interstitial fluid. The interstitial fluid contains water, ions and other small molecules. Such a fluid corresponds to plasma without macromolecules and interacts with the ground substance, thereby forming a viscous hydrated gel that can stabilize fiber network [29,30].
2. The Darcy law is used to approximate fibers of the media as a continuum and allows us to compute the actual microscopic flow phenomena that occur in the fibrous media.
3. Brinkman's law then allows us to describe the flow field around solid bodies such as the embedded cells in extracellular matrix.
4. Transient convective Brinkman's equations [31–33] are applied to simulate interstitial flow in a fibrous medium driven by a moving needle [19].

Although the previously stated approach cannot give information on microscopic events, it can describe macroscale flow patterns in porous media. Focus is given to the effects of interstitial fluid flow during implantation of an acupuncture needle until the tip has reached the desired location within the hypodermis. The objective of this work is to give a description of the physical stress (shear stress and pressure) influencing tissue and cells.

2. Methods

On a microscopic scale, the interstitial tissues are composed of fluid, cells, and solid fibers. The interstitial fluid contains water, ions and other small molecules. Such a fluid corresponds to plasma without macromolecules [22]. It interacts with the ground substance to form a gel-like medium.

A model taking into account individual fibers and cell adhesion complexes is already a falsification of the reality. Moreover, it is very costly from the computational viewpoint. When considering an organized homogeneous matrix of fibers, computation of such a model shows the microscopic fluctuations of the fluid shear stress at the protein level [34].

Due to biological complexity, the interstitium is considered as a fluid-filled porous material [22]. The interstitial flow is simulated using the incompressible convective Brinkman equation. The phenomenological model cannot give information on unneeded microscopic events but the Darcy equation can describe macroscale flow patterns in porous media.

2.1. Flow equations

The governing equations of the unsteady flow of an incompressible fluid through a porous medium (with mass density ρ, dynamic viscosity μ, and kinematic viscosity $\nu = \mu/\rho$) can be derived as [31–33]:

$$\frac{\rho}{\alpha_f}\left(\frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla\left(\frac{\bar{\mathbf{u}}}{\alpha_f}\right)\right) - \mu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\alpha_f}\nabla(\alpha_f p_f) = -\frac{\mu}{P}\bar{\mathbf{u}} \quad \text{in } \Omega(t), \tag{1}$$

$$\nabla \cdot \bar{\mathbf{u}} = 0, \tag{2}$$

$$\bar{\mathbf{u}}(\mathbf{x}, 0) = \bar{\mathbf{u}}_0(\mathbf{x}), \tag{3}$$

where $-\frac{\mu}{P}\bar{\mathbf{u}}$ denotes the Darcy drag, P the Darcy permeability, $\bar{\mathbf{u}}$ the averaged velocity vector, and p_f the pressure. The averaged velocity is defined as

$$\bar{\mathbf{u}} = \alpha_f \mathbf{u}_f, \tag{4}$$

where $\mathbf{u}_f$ is the fluid velocity and

$$\alpha_f = \frac{\text{fluid volume}}{\text{total volume}} \tag{5}$$

is the fluid volume fraction. This volume fraction corresponds to the effective porosity of the medium. The fluid fractional volume α_f is taken as a space-dependent parameter to model the distinguished properties around an acupoint.

The system of equations (1–2) is applied to the case of a flow driven by the motion of a needle in the deformable domain $\Omega(t)$ [19]. The domain boundary can be decomposed into four surfaces: the needle boundary denoted by Γ_1, an impervious boundary (wall) denoted by Γ_2, the mastocyte membrane denoted by Γ_3, and the open boundary on the sides denoted by Γ_4. The classical no-slip condition is applied to the needle surface Γ_1, the rigid wall Γ_2, and the cell surface Γ_3. At the outer boundary Γ_4 a traction-free boundary condition is prescribed. Thus, the entire set of boundary conditions reads as

$$\bar{\mathbf{u}} = \mathbf{v}_{\text{needle}}, \ \text{ on } \Gamma_1, \tag{6}$$

$$\bar{\mathbf{u}} = 0, \ \text{ on } \Gamma_2, \tag{7}$$

$$\bar{\mathbf{u}} = 0, \ \text{ on } \Gamma_3, \tag{8}$$

$$-\mu \nabla \bar{\mathbf{u}} \cdot \mathbf{n} + p_f \mathbf{n} = 0, \ \text{ on } \Gamma_4. \tag{9}$$

2.2. Finite element model

The governing equations in Section 2.2.1 are solved using the finite element software FreeFem++ [35]. This code programs the discrete equations derived from the finite element weak formulation of the problem presented in Section 2.2.3 using a characteristic/Galerkin model to stabilize convection terms.