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Monotone level-sets on arbitrary meshes without redistancing

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ABSTRACT

In this paper we present approaches that address two issues that can occur when the level-set method is used to simulate two-fluid flows in engineering practice. The first issue concerns regularizing the Heaviside function on arbitrary meshes. We show that the regularized Heaviside function can be non-smooth on non-uniform meshes. Alternative regularizing definitions that are indeed smooth and monotonic, are introduced. These new definitions lead to smooth Heaviside functions by taking the changing local mesh-size into account. The second issue is the computational cost and fragility caused by the necessity of redistancing the level-set field. In [1, 2] it is shown that strongly coupling the level-set convection with the flow solver provides robustness and potentially efficiency and accuracy advantages. The next step would be to include redistancing within the strong coupling part of the algorithm. The computational cost of current redistancing procedure prohibit this. Four alternative approaches for circumventing the expensive redistancing step are proposed. This should facilitate a fully coupled level-set approach. Some benchmark cases demonstrate the efficacy of the proposed approaches. These includes the standard test case of the vortex in a box. Based on these results the most favourable redistancing approach is selected.

1. Introduction

Level-sets are a very powerful approach for solving interface problems [3]. By disconnecting the interface description from the underlying mesh, topology changes of the interface are handled with ease. For numerical reasons, such as numerical quadrature or finite differencing, the interface is often given a finite mesh-dependent width. This is done by adopting a so-called regularized Heaviside function, where the transition from zero to unity does not occur instantaneously but over a finite band. This regularization, however, causes two problems. On irregular meshes – which might occur in the analysis of engineering artefacts – the smoothed Heaviside can become non-smooth or even non-monotone. Additionally, the level-set is required to be a distance function in order to control the thickness of the interface. Due to its evolution the level-set evolves requires to be redistanced, that is the distance property needs to be actively repaired.

In [1,2] the level-set method is used to model a rigid-body floating on a water surface. This water surface is handled using a level-set. In this paper we adopted a strongly coupled modeling approach. Meaning that both the flow; interface evolution; body motion and mesh deformation are solved simultaneously. In agreement with [4–6] – and other FSI literature– this approach results in a robust solution strategy. In a pure two-fluid problem, i.e. without a floating object, the strongly coupled approach proved to be more robust than traditional approaches where interface evolution is solved after the flow problem is solved.

However, the redistancing of the level-set was not included in the strongly coupled part of the solver, this was done at the end of each time step. Due to the redistribution of mass associated with redistancing, momentum and energy conservation are difficult to control. Certainly, the energy errors are troublesome as these might trigger instabilities in the solver. Therefore, there is a desire to include the redistancing step in the strongly coupled part of the method. This should allow methods that are either energy conservative or guaranteed to be energy dissipative. This is envisioned to lead to more robust and more accurate methods.

In this paper we present four alternative approaches which lead to an approximate distance field without directly solving a redistancing problem. These approaches are formulated in such a way that they potentially solve the non-smoothness on irregular grids.

The paper is structured as follows, in Section 2 provides a short introduction to the level-set method and the regularized Heaviside function. In Section 3 we show that a naive definition of the regularized Heaviside can actually be non-smooth. A potential solution to this problem is presented.

In Section 4 we build on the result from the previous section and introduce four alternative approaches to redistance the level-set field. All these approaches circumvent solving the difficult non-

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linear Eikonal equation. The Eikonal equation is translated to simple projection problems that achieve the same: a distance field suitable to define a smooth Heaviside.

In Section 5 we give a short description of the numerical formulations and the finite element and isogeometric discretizations employed to solve the convection and redistancing problems in the next section. In this section, Section 6, we first solve a simple redistancing problem on irregular grids and then solve the vortex in a box problem using different discretizations and different redistancing approaches. Using these test cases we identify the best method, and the appropriate numerical parameters. For this method a mesh convergence study is performed and suitability in three dimensions is tested.

In Section 7 we conclude and sketch a perspective for further use of the proposed methods

2. The level set method

The level-set method has since its origin [3] been applied to numerous problems involving surfaces, interfaces and shapes. Its application can be found in a wide range of areas from image processing, computer graphics, topology and shape optimisation and simulation of physics problems at interfaces such as crystal growth, or two-fluid flow.

See, the review papers [7,8] or books [9,10] for a broad picture of the applications and methods available. In this paper we focus on level-set in combination with variational methods such as finite element and isogeometric analysis [11] with the final goal of applying it to two-fluid flow. The two-fluid problem is a physical problem where the interface between the fluids is unknown and physical parameters are discontinuous across the interface. Adoption of a level-set allows for easy handling of interface topology changes and regularisation of the discontinuity.

2.1. Level-sets for two-fluid problems

In the level-set method a surface of lower dimension, such as an interface between two distinct materials is indirectly parameterized employing a globally defined function. This function is denoted as ϕ, and defines a surface as follows,

$$\Gamma_i = \{\boldsymbol{x} \in \Omega : \phi = 0\}. \tag{1}$$

This automatically leads to the following distinct subdomains,

$$\begin{aligned} \Omega^- &= \{\boldsymbol{x} \in \Omega : \phi < 0\}, \\ \Omega^+ &= \{\boldsymbol{x} \in \Omega : \phi > 0\}, \end{aligned} \tag{2}$$

which allows the prescription of different physical parameters in each subdomain. For instance, the density

$$\rho = \begin{cases} \rho_0 & \text{if} \quad \boldsymbol{x} \in \Omega^-, \\ \rho_1 & \text{if} \quad \boldsymbol{x} \in \Omega^+. \end{cases} \tag{3}$$

Alternatively, the Heaviside function

$$H(\phi) = \begin{cases} 0 & \text{if} \quad \phi < 0, \\ \frac{1}{2} & \text{if} \quad \phi = 0, \\ 1 & \text{if} \quad \phi > 0, \end{cases} \tag{4}$$

can be used for a convex interpolation to define a density

$$\rho = \rho_0(1 - H(\phi)) + \rho_1 H(\phi). \tag{5}$$

This has the advantage of automatically handling the interface itself in a natural way.

2.2. Regularized heaviside

In numerical methods the sharp interfaces defined in the previous section can lead to problems. For instance this appears when determining a mass matrix, M_{ab} is approximated by quadrature as,

$$M_{ab} = \int_\Omega \rho N_a N_b d\Omega \approx \sum_{i=1..n_{ip}} \rho(\boldsymbol{x}_i) N_a(\boldsymbol{x}_i) N_b(\boldsymbol{x}_i) w_i. \tag{6}$$

A sudden change of the density leads to a very bad approximation of the intended integral.

To alleviate this problem the sharp Heaviside function, defined in Eq. (4), is replaced by a regularized Heaviside function. This regularized Heaviside function is often defined as

$$H_\epsilon(\phi) = \begin{cases} 0 & \text{if} \quad \phi < -\epsilon, \\ \frac{1}{2}(1 + \sin(\frac{\pi\phi}{2\epsilon})) & \text{if} \quad \phi = 0, \\ 1 & \text{if} \quad \phi > \epsilon, \end{cases} \tag{7}$$

where ϵ is the smoothing distance. Instead of an instantaneous switch from 0 to 1, this switch is spread over a finite layer around the interface. To have strict control over the width of this interface layer, we require ϕ, to be a signed distance function. This means it needs to satisfy

$$\|\nabla\phi\| = 1, \tag{8}$$

which is the Eikonal equation. As the regularizing is introduced to deal with numerical issues, such as quadrature, it is natural to specify the finite interface layer in terms of mesh size h as

$$\epsilon = \alpha h. \tag{9}$$

Here α is an $O(1)$ parameter. The meshsize can be defined unambiguously for structured equidistant meshes. However, on arbitrary meshes this is not always straightforward. In [1,12,13] we employed the meshsize

$$h = \frac{\|\nabla\phi\|}{\sqrt{\nabla\phi \cdot \boldsymbol{G}\nabla\phi}}, \tag{10}$$

where $\boldsymbol{G}$ is the metric-tensor

$$\boldsymbol{G} = \left(\frac{\partial\xi}{\partial\boldsymbol{x}}\right)^T \frac{\partial\xi}{\partial\boldsymbol{x}} \tag{11}$$

where $\boldsymbol{x}$ is the physical space coordinate and ξ is the coordinate in parametric space pertaining to the reference element.

This definition of h incorporates the desired directional information. Effectively, a length scale is extracted from the metric-tensor in the direction $\frac{\nabla\phi}{\|\nabla\phi\|}$.

3. Monotonicity on arbitrary meshes

In this section we further discuss the smoothing of the Heaviside function on arbitrary meshes. For this exposition it is useful to slightly rewrite the regularized Heaviside function,

$$\hat{H}(\hat{\phi}) = \begin{cases} 0 & \text{if} \quad \hat{\phi} < -\alpha, \\ \frac{1}{2}(1 + \sin(\frac{\pi}{2}\frac{\hat{\phi}}{\alpha})) & \text{if} \quad \hat{\phi} = 0, \\ 1 & \text{if} \quad \hat{\phi} > \alpha, \end{cases} \tag{12}$$

where

$$\hat{\phi} = \frac{\phi}{h} \tag{13}$$

is the scaled distance. In other words, if ϕ is the *actual* distance to the interface, expressed in for instance *mm* or *m*, then $\hat{\phi}$ can be thought of as that same distance but expressed in *number of elements*, that is a multiple of a typical element length. However, since this scaling occurs locally, the variation of h along the path– from interface to the point under consideration – is not taken into account. Therefore, this rescaled distance is only an effective estimate if h varies only mildly (or not at all) across the interface.