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# Investigating the effect of micro-structure on the deformation of saturated fibrous media using direct numerical simulations

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ABSTRACT

To understand the effect of micro-structure on the deformation of saturated fibrous media, this study investigates the viability of representing real layered fibrous media through model geometry made of cylinders in orthogonal arrangements. Direct numerical simulations of compression performed on such model geometries, using a coupled lattice Boltzmann and finite element method, facilitate the investigation of micro-structure effect on deformation.

The deformational response of the model cylindrical arrangements representing porous media is validated with an existing analytical solution and in doing so the bulk elastic modulus of such an arrangement is evaluated. Based on the critical parameters porosity, permeability and compressive modulus, it is found that cylinders in staggered orthogonal arrangement behave as real layered fibrous porous media during saturated compression. Furthermore an analytical expression is developed to predict the compressive modulus of orthogonal arrangement of cylinders. The expression shows that fiber diameter does not affect the compression of such fibrous media which only depend on void fraction. This is further confirmed through direct numerical simulations.



## 1. Introduction

Non-woven fabrics are a rapidly growing sector in textile industry. These layered fibrous porous media find use in filtration, absorbency, de-watering and packaging industries. In the paper industry, felt a non-woven fibrous porous media is used in de-watering wet paper. Fig. 1 shows the schematic of the wet-pressing process where water is squeezed out of the paper and felt system as they pass through the rollers. Improving the de-watering efficiency implies increasing compressibility and decreasing water retention in felts. Similarly a number of cleaning products made from non-woven material require specific characteristics of compressibility and water retention to meet the application demands. In order to develop newer designs in these fibrous products it is critical to understand the effect of micro-structure on the deformational characteristics of these media.

The existing techniques for modeling deformable saturated porous media such as the Biot's Theory (BT) [5–8] and the Theory of Porous Media (TPM) [10–12,16] are based on homogenization principles which assume that all the phases co-exist at a physical location. Thus, such techniques are incapable of performing micro-mechanical investigations to investigate the effect of micro-structure on bulk properties. Techniques based on mixture theory have been applied, for instance by Kataja et al. [22], Lewalle et al. [27] and Bezanovic et al. [4] to study paper pressing assuming the paper and felt to be homogeneous porous media. More recently, Khoei and Haghighat [25] developed a numerical scheme that also uses mixture theory to approximate variables over a finite element to solve the deformation of saturated porous media.

The stress-strain relationship in fibrous porous media has been studied for application in proton exchange membrane (PEM) fuel cells, where the stresses in the porous gas diffusion layer (GDL) can impact the performance of the fuel cell [17]. Analytical models have also been developed based on beam theory that can predict the compression of the fibrous GDL at strain values greater than 0.5 [18,30]. However these studies did not attempt to investigate the effect of micro-structure on the deformation of the fibrous media. Moreover they did not include the deformation due to contact in their modeling work.

The authors previously showed [24] that the deformational response of a generic porous medium can be recovered using regular arrangements of simple geometries, such as spheres in simple cubic arrangement, provided that the parameters porosity ($n^F$), permeability ($K^F$) and the average (secant) bulk elastic modulus ($E_{avg}$) of the generic porous medium and the model geometry are the

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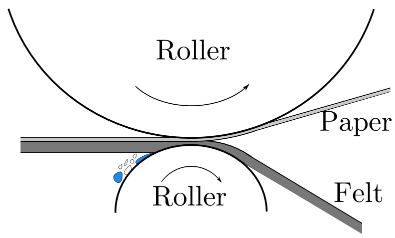


**Fig. 1.** Water removal from paper through the wet-pressing of paper and felt system.

same. The investigations are limited to fibrous media in this work, where the deformation mechanism involves both deformation due to contact and due to bending, unlike in granular media. Therefore, the previous work by the authors is extended to geometries made up of cylinders in orthogonal arrangement. Particularly, this would allow to validate the contact model that depends on the geometry of the solid surface. In doing so, an appropriate method to determine the tangent bulk modulus ($E_b$) of cylindrical arrangements is obtained, which is used later to evaluate the tangent bulk modulus of more complicated geometries.

Investigating the micro-structure affect in fibrous media using modeling techniques can become complicated and tedious if it involves generating geometry from X-ray tomography data, meshing such a geometry and then running expensive computer simulations. Therefore, in this work an alternative approach was adopted. By evaluating the relationship between $n^F$, $K^F$ and $E_b$ for two model geometries consisting of orthogonal arrangement of cylinders, it was found that the "Staggered" (also referred to as "skewed") orthogonal arrangement of cylinders behaves like a real fibrous media under compression. Therefore, the staggered orthogonal arrangement of cylinders was used as the model geometry to investigate the effect of fiber diameter on the compression of fibrous media under the application of external load. A combination of theoretical and numerical analysis (using LB-FE method) is employed for the work.

## 2. Methodology

The parallel hybrid Lattice Boltzmann and finite element (LB-FE) method is employed to carry-out direct numerical simulations of fluid-structure interaction in deforming fibrous media. Detailed description of the numerical scheme, parallelization methodology, validation and grid refinement studies can be found in the previous work of the authors [23,24]. For completeness, a brief description of the method is provided below.

The single relaxation D3Q19 implementation [1,2] of lattice Boltzmann method is used to model the fluid phase. The accuracy of LBM has been compared to traditional numerical methods in fluid dynamics such as finite difference and finite volume methods [3,32]. Moreover, the local nature of the calculations and the simple bounce-back scheme for no-slip condition make LBM a favorable tool for modeling single-phase flow through porous media [29,31], where the inherent parallel nature of LBM can be leveraged through distributed computing.

In order to model the solid deformations, a linear elastic finite element method is used. The four-node tetrahedral element is used to discretize the geometry and solve the weak form of the Cauchy's equation. The Newmark's scheme is used to perform time integration.

The transfer of momentum between the fluid and solid phases is effected through the popular "link-bounce-back" scheme [1,2,26].

A near contact model developed by Ding and Aidun [15] and later modified for elastic finite element particles by MacMeccan et al. [28] is used in this work. A link-wise repulsive force, based on the lubrication and contact forces, is applied to the interacting surfaces. The magnitude and direction of the link-wise force is given as

$$\delta \mathbf{F}_k = \begin{cases} 0 & \text{if } \; s > c_k, \\ \frac{3q}{2c_k^2\lambda}\nu^F\rho^F\left(\frac{1}{s^2}-\frac{1}{c_k^2}\right)\mathbf{U_a}\cdot\mathbf{e_k} & s_c < s < c_k, \\ \frac{3q}{2c_k^2\lambda}\nu^F\rho^F\left(\frac{1}{s^2}-\frac{1}{c_k^2}\right)\mathbf{U_a}\cdot\mathbf{e_k} + A_c\exp\left(\frac{-s+s_c}{\sigma_c}\right) & 0 < s < s_c, \end{cases} \tag{1}$$

where $\mathbf{U_a}$ is the approach velocity of the interacting surfaces, $\mathbf{e_k}$ is the direction vector of the lattice link and $\nu^F$, $\rho^F$ are the kinematic viscosity and density of the fluid. $c_k$ is the length of a link, $s$ is the link-wise gap between surfaces, $s_c$ is the contact cutoff distance, $\lambda$ is the local surface curvature and $q$ is chosen to be 0.6 [15]. The parameters $s_c$ and $\sigma_c$ are dependent on the surface roughness of the geometry and are determined *a priori*. $A_c$ is chosen such that the repulsive contact force scales appropriately with the applied stresses ($\sigma$) i.e. $A_c \approx \frac{\sigma}{a_o}$ where $a_o$ is the contact area that varies with the applied stress. For non-conformal, convex bodies, $a_o$ can be determined based on $\sigma$ and surface curvature.

Khan and Aidun [24] validated the contact model in the LB-FE implementation with Hertzian contact model for contact deformation in spheres and cylinders. They found that for small deformations, the implemented contact model follows the Hertzian predictions. Additionally using direct numerical simulations Khan and Aidun [24] were able to recover the deformation response of a real porous media as predicted by the analytical solution of de Boer et al. [9]. They employed an idealized porous medium made up of spheres in a simple cubic arrangement. However the ability of the numerical method to accurately simulate the deformational response of fibrous media has not been tested.

Grid refinement studies were carried out for both the lattice Boltzmann grid [23] and the finite element grid [24]. Due to the presence of two different grids an additional parameter $l_{fea}$ is introduced. $l_{fea}$ is defined as the ratio of finite element grid size to the lattice Boltzmann grid size. A linear interpolation scheme is chosen to transfer momentum between the two phases, thus it was found that a value of $l_{fea} = 2.0$ was necessary and sufficient [23,28]. The grid refinement studies showed that a lattice Boltzmann grid with 40 lattice units across the sphere diameter would accurately capture the drag force on the sphere and a finite element grid with 10 to 12 elements across the fiber diameter can model the small strain deformation accurately. Fig. 2 shows the typical LBM and FEA mesh used in the fluid-structure interaction simulations carried out in this work.

## 3. Behavior of ordered fibrous porous media

In this section, the behavior of saturated porous media under compressive loading is analyzed using model geometry made up of cylindrical arrangements. de Boer et al. [9] provided the analytical solution for the deformation of a semi-infinite saturated porous medium under an applied load with no body force term. They assumed small strain, thus neglecting any variation in volume fractions. The resulting response of solid displacement $u_z(z, t)$ is given as

$$u_z(z,t) = -\frac{1}{\sqrt{\mathfrak{a}}(\lambda^S + 2\mu^S)}\int_0^t \times\left[f(t-\tau)e^{-\frac{\mathfrak{b}}{2\mathfrak{a}}\tau}I_0\left(\frac{\mathfrak{b}\sqrt{\tau^2-\mathfrak{a}z^2}}{2\mathfrak{a}}\right)U\left(\tau-\sqrt{\mathfrak{a}}z\right)\right]d\tau \tag{2}$$