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Radial flows in heterogeneous porous media with a linear injection scheme

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ABSTRACT

A robust diffuse-interface formulation is applied to simulate miscible and immiscible radial flows in heterogeneous porous media, in which the permeability is characterized by a log Gaussian distribution. The stabilizing effects of linear injection scheme, determined by the fingering interfacial length, are investigated to verify its applicability in various conditions of miscibility and permeability heterogeneity. For fully miscible conditions, the linear injection scheme shows destabilizing effects both in homogeneous and heterogeneous media. On the other hand, even the immiscible fingering instability in a homogenous medium can be suppressed effectively by applying the linear injection scheme, the stabilizing effect on such a linear injection scheme is insignificant in a heterogeneous medium.

1. Introduction

Viscous fingering of the Saffman–Taylor instability [1] arises when a more viscous fluid is displaced by another fluid of lower viscosity in a porous medium or Hele-Shaw cell. Particularly, the viscous fingering problem driven by radial injection had been investigated for several decades [2], because of its important applications to enhanced oil recovery [3–5]. In oil recovery processes, once the actively evolving branched fingers of injected less viscous fluid reach the production well, only insignificant amount of the more viscous crude oils can be further retrieved. To improve the oil recovery efficiency, recent efforts have been focused on effectively controlling the fingering patterns in a homogeneous porous media (or Hele-Shaw cell) by time-dependent injection schemes [6–11]. A strategy of time-dependent injection scheme for miscible fluids is designed by variant injection strength scaled with time like $t^{-1/3}$ [6]. On the other hand, several investigations tackled the problem with immiscible fluids in a similar exponentially time-dependent injection scheme [7–9]. Under these time-dependent injection schemes, the traditional multi-branched patterns are significantly constrained. Furthermore, an optimal linear injection scheme is proposed in an immiscible condition to minimize the growth of interfacial amplitudes [10]. This optimal linear injection scheme has been generally verified for immiscible or partially miscible cases, in which surface tension or effective interfacial tension (or the so-called Korteweg stress) respectively plays an important role in stabilization [11]. Nevertheless, the stabilizing effects are not conclusive in a miscible interface because of vigorous secondary fingering phenomena, e.g., merging and tip-split, which might be triggered immediately after the injection [11].

These time-dependent injection schemes are all developed in a homogeneous porous medium, i.e., Hele-Shaw cell. However, the heterogeneous distribution of permeability in many porous environments in practical applications, e.g., reservoir rocks, may strongly affect the fingering instability. For instance, the flow shows preferred path due to the permeability heterogeneity, and might result in undesired faster arrivals of the injected fluid to reach the production well [12–14]. In addition, simulations show the flow paths are dictated by the permeability field for sufficiently strong heterogeneity [15–20]. An interesting question arises if these time-dependent scheme coupled with the permeability heterogeneity, particularly the optimal linear injection scheme, can still effectively constrain the emergence of fingering instability in different miscibility conditions? These can be numerically achieved by a diffuse-interface approach of the Darcy–Cahn–Hilliard model [20], an analogy of the so-called Hele-Shaw–Cahn–Hilliard equations [11,21–25]. By properly choosing profiles of the interfacial free energy, this diffuse-interface formulation is capable of dealing with immiscible and miscible interfaces. In the present study, we focus on the fingering patterns induced by the linear injection scheme in a heterogeneous porous medium. In addition, quantitative measures will be presented to determine the applicability of such a linear injection scheme to fingering control. This paper consists of three additional sections. The theoretical background, and the numerical methods are introduced in Section 2. Numerical results and conclusions are discussed and summarized in Section 3 and Section 4, respectively.

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2. Physical problem

2.1. Governing equations

We consider a two-dimensional porous medium with heterogeneous permeability $k(x, y)$. A less viscous fluid 1 (viscosity η_1) is injected to displace another more viscous fluid 2 (viscosity η_2) originally occupying the medium. These two incompressible fluids can be either immiscible or fully miscible to each other. Initially, the fluid–fluid interface is a small circular core of diameter D_0, and a Cartesian coordinate system (x, y) is defined in such a way that its origin is located at the center of this core region. The less viscous fluid is injected at a point source located at the origin. The process continues up to a time $t = t_f$, when the area of the injected fluid expands to $\pi D_f^2/4$ in a stable injection condition without fingering instability. The injection strength $Q(t)$, equal to the area covered per unit depth, follows a linear injection rate proposed in Refs. [10,11], so that injection rate is given by $Q(t) = \pi(D_f - D_0)(D_0 t_f + (D_f - D_0)t)/2{t_f}^2$. Driven by the action of injection, as time progresses the interface becomes unstable. The governing equations for a diffuse-interface approach based on the Darcy–Cahn–Hilliard , or Hele-Shaw–Cahn–Hilliard, model can be written as [21,24,25]

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\nabla p = -\frac{\eta}{k}\mathbf{u} - \epsilon\rho\nabla \cdot \left[(\nabla c)(\nabla c)^T\right], \tag{2}$$

$$\rho\left(\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c\right) = \alpha\nabla^2\mu, \tag{3}$$

$$\mu = \frac{\partial f_0}{\partial c} - \epsilon\nabla^2 c. \tag{4}$$

Here, $\mathbf{u}$, p, ρ and η denote the velocity vector, the pressure, the density and the viscosity, respectively. The phase-field variables c of the injected and the surrounding displaced fluids are set as $c = 1$ and $c = 0$, respectively. The constant ϵ represents the coefficient of capillary, while the constant α denotes the coefficient of mobility. The chemical potential is μ, and f_0 is the classical part of the free energy (or the Helmholtz free energy).

In this diffuse-interface framework, the viscosity (η) correlation of fluids is assumed to be related to the phase-field variable (c) determined by a constant parameter R as [4,17]

$$\eta(c) = \eta_1 e^{[R(1-c)]}, \quad R = \ln\left(\frac{\eta_2}{\eta_1}\right). \tag{5}$$

Permeability fields $k(x, y)$ associated with desired statistical distribution are expressed in terms of a characteristic value K and random function $g(x, y)$, whose Gaussian distribution is characterized by the variance s and the spatial correlation scale l. An algorithm originally provided by Shinozuka and Jen [26] is employed to generate the permeability distributions [15–17,20], which is described by

$$k(x, y) = Ke^{g(x,y)}, \tag{6}$$

$$g(x, y) = s^2 exp\left(-\pi\left[\left(\frac{x}{l}\right)^2 + \left(\frac{y}{l}\right)^2\right]\right). \tag{7}$$

We follow the undimensional processes applied in Ref. [20], such that D_f and t_f are taken as the characteristic scales. Furthermore, the pressure and the free energy are scaled by $(\eta_1 D_f^2)/(Kt_f)$ and a characteristic specific energy f^*, respectively. Thus, the dimensionless versions of the governing equations are

$$\nabla \cdot \mathbf{u} = 0, \tag{8}$$

$$\nabla p = -\frac{\eta}{k}\mathbf{u} - \frac{C}{I}\nabla \cdot \left[(\nabla c)(\nabla c)^T\right], \tag{9}$$

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = \frac{1}{\mathrm{Pe}}\nabla^2\mu, \tag{10}$$

$$\mu = \frac{\partial f_0}{\partial c} - C\nabla^2 c. \tag{11}$$

Dimensionless parameters, such as the Atwood number A (normalized viscosity contrast), the Péclet number Pe, the Cahn number C, and the injection strength I are defined as

$$\mathrm{Pe} = \frac{\rho D_f^2}{\alpha f^* t_f}, \quad A = \frac{e^R - 1}{e^R + 1}, \quad C = \frac{\epsilon}{D_f^2 f^*}, \quad I = \frac{\eta_1 D_f^2}{\rho f^* K t_f}.$$

The diffusional Péclet number and the Cahn number are the non-dimensional measures of the dissipation and dispersion in the model [27].

Profiles of the free energy govern miscibility of the fluid interface [11,20,21]. To separate the fluid phases for an immiscible interface, a concave profile of the free energy should be prescribed. The dimensionless expression applied in Refs. [24,25] is used to simulate the condition of two immiscible fluids as

$$f_0 = c^2(1-c)^2. \tag{12}$$

By this formulation, the dimensionless surface tension, denoted as σ, on the immiscible interface associated with a given spatial variable ζ can be expressed as [11,25]

$$\sigma = \frac{1}{I}\int\left[f_0 + \frac{C}{2}\left(\frac{\partial c}{\partial \zeta}\right)^2\right]d\zeta. \tag{13}$$

On the other hand, the phases are allowed to mix if a convex free energy profile is applied [11,20,21], which is suitable for a miscible interface. To include the diffusion between two miscible fluids, we prescribe the dimensionless free energy as

$$f_0 = \frac{1}{2}\left(c - \frac{1}{2}\right)^2. \tag{14}$$

By further assuming $C = 0$, the phase variable equations, e.g., Eqs. (10) and (11), converge to the conventional advection–diffusion equation as

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = \frac{1}{\mathrm{Pe}}\nabla^2 c. \tag{15}$$

2.2. Numerical schemes

The numerical methods we employ in this work are similar to the ones developed in Refs. [4,11,17,20,24], in which the governing equations are reformulated into the well known streamfunction (ϕ)–vorticity (ω) system, and yield

$$u = \frac{\partial\phi}{\partial y}, \quad v = -\frac{\partial\phi}{\partial x} \tag{16}$$

$$\nabla^2\phi = -\omega, \tag{17}$$

$$\omega = -R\left(u\frac{\partial c}{\partial y} - v\frac{\partial c}{\partial x}\right) - \frac{1}{k}\left(u\frac{\partial k}{\partial y} - v\frac{\partial k}{\partial x}\right) + \frac{k}{\eta}\frac{C}{I}\left[\frac{\partial c}{\partial x}\left(\frac{\partial^3 c}{\partial x^2\partial y} + \frac{\partial^3 c}{\partial y^3}\right) - \frac{\partial c}{\partial y}\left(\frac{\partial^3 c}{\partial x\partial y^2} + \frac{\partial^3 c}{\partial x^3}\right)\right]. \tag{18}$$

Since presence of the point source, which involves singularity at the origin, the total velocity is divided into two components, the rotational ($\mathbf{u}_{rot}$) and potential ($\mathbf{u}_{pot}$) parts. The rotational part of the velocity, induced by viscosity contrast and heterogeneity, is obtained numerically. The potential radial velocity by injection is smoothed out