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The intermittent nature of the laminar separation bubble on a cylinder in uniform flow

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ABSTRACT

Flow past a circular cylinder in a uniform flow is investigated for $1 \times 10^4 \leq Re \leq 4 \times 10^5$. A stabilized finite element method is used to solve the incompressible flow equations in primitive variables in three dimensions. The computations capture the phenomenon of drag crisis: a significant reduction in drag with increase in *Re* in the critical regime. The mechanism for this decrease in drag during the drag crisis is explored. It is found that the transition of the boundary layer from a laminar to turbulent state, as well as the formation of the laminar separation bubble (LSB), is intermittent. The LSB does not exist in the sub-critical regime while it appears at all times beyond the critical regime. The frequency of its appearance as well as the duration of its stay, in the critical *Re* regime, increases with increase in *Re*. This is established by studying the *rms* of the high pass filtered fluctuations in the surface pressure and flow close to the cylinder. A procedure to capture these fluctuations due to the shear layer activity responsible for the formation of LSB is proposed and implemented. It is utilized to estimate the intermittency factor of the LSB at various *Re*. It is found that the intermittency can be utilized to explain the variation of mean drag with *Re*. Weakening of vortex shedding is observed in critical regime and beyond.

1. Introduction

The flow past a circular cylinder is associated with very rich phenomena and has enormous scientific and engineering significance. Consequently, it has been a subject of extensive research both experimentally and numerically. Williamson [24] presented a comprehensive review of the flow at various Reynolds number. The Reynolds number, *Re*, is based on the diameter of the cylinder, free stream speed and the coefficient of kinematic viscosity of the fluid. The flow is extremely complex, especially near the critical *Re*, where the boundary layer transitions from a laminar to turbulent state. In this regime, the laminar boundary separates as it negotiates the shoulder of the cylinder, transitions to a turbulent state and reattaches, leading to the formation of a laminar separation bubble (LSB). The turbulent boundary layer separates further downstream compared to the laminar boundary layer. As a result, the wake is narrower and the base pressure higher. This leads to a very rapid reduction in the drag coefficient ($\overline{C}_D$) with increase in Reynolds number and is referred to as *drag crisis*. Roshko [15] identified four regimes of the flow based on the state of the boundary layer: (i) sub-critical for $Re < 2 \times 10^5$, (ii) critical for $2 \times 10^5 \leq Re \leq 5 \times 10^5$, (iii) supercritical for $5 \times 10^5 < Re < 5 \times 10^6$ and (iv) transcritical for $Re > 5 \times 10^6$. In the subcritical regime the boundary layer undergoes laminar separation and the mean drag coefficient ($\overline{C}_D$) remains almost constant with increase in *Re*. The wake is relatively wide. In the critical regime, the boundary layer is laminar when it first separates from the surface of the cylinder. The separated shear layer undergoes a transition to a turbulent state and reattaches to the surface of the cylinder (Behara and Mittal [5], Singh and Mittal [19]). It separates as a turbulent boundary layer further downstream from the surface of the cylinder. A decrease in $\overline{C}_D$ with increase in *Re* is observed in this regime. In the supercritical regime the $\overline{C}_D$ increases with increase in *Re*. It remains nearly constant with *Re* in the transcritical regime. Achenbach [1] carried out experiments on smooth cylinder and provided more insight on these regimes. With the help of skin friction and pressure distribution on the surface of the cylinder, the presence of a laminar separation bubble (LSB) was surmised in the critical regime.

In another experimental investigation, Bearman [3] found that the variation of $\overline{C}_D$ with *Re* during the drag crisis, in the critical regime, takes place in two stages. In the first stage, at the lower *Re* of the critical regime, the LSB occurs close to only one of the two shoulders of the cylinder. At higher *Re*, the LSB appears on both sides of the cylinder. This was supported by studies carried out by Schewe [17] who also showed that the boundary layer transition first occurs only on one side of the cylinder. In this state, the cylin-

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der experiences a non-zero mean lift. Behara and Mittal [5] carried out three-dimensional large eddy simulations (LES) for smooth cylinder and with a trip located at $\theta = 55^\circ$ from the stagnation point on one of the sides. They observed that the smooth cylinder undergoes a single stage drag crisis while the one with a trip experiences a two-staged drag crisis similar to what is seen in experimental studies. This suggests that the asymmetry in the transition of the flow on the two sides of the cylinder is a consequence of a slight asymmetry in either the geometry or the flow conditions at the inlet. Cadot et al. [7] reported two kinds of boundary layer reattachment during the critical regime: symmetric and asymmetric. In the asymmetric scenario two kinds of transition are possible. In the first one, the flow randomly explores three states: no reattachment, reattachment only on one side, and reattachment only on the other side. The other possible transition involves bistable states. In the symmetric scenario, the boundary layer reattachment occurs simultaneously on both sides of the cylinder. Weakening of vortex shedding activity was also observed during drag crisis. All the analysis was carried out using time histories of the surface pressure. The probability of appearance of one and two bubble states was presented for various scenarios of transition. It was observed that for asymmetric reattachment, one bubble state has a higher probability at the onset of the critical regime, while the two bubble state shows increased probability at *Re* close to the end of critical regime. In an experimental study, Miau et al. [13] found that the development of LSB causes the vortex shedding frequency to fluctuate with large uncertainty. Lehmkuhl et al. [12] performed three-dimensional large eddy simulations with high span wise grid resolution. They observed asymmetric reattachment in the critical regime.

Tani [20] presented a review of the low speed flows involving laminar separation bubbles on airfoils and cylinder. The experimental measurements by Fage and Falkner [8] were also reviewed and discussed. The presence of LSB in the critical regime was identified via pressure distribution and skin friction over the surface of the cylinder. Similar observations were made by Lehmkuhl et al. [12]. Singh and Mittal [19] investigated this regime by carrying out two-dimensional simulations. They reported that the interaction between the shear layer instability and boundary layer leads to transition to turbulent flow and the formation of LSB.

Despite the various studies in the critical regime, the exact dynamics of the formation of the LSB on the circular cylinder is not well understood. Does the LSB appear suddenly at a certain critical *Re*? Or does it have an intermittent nature at the lower *Re* end of the critical regime and exists at all times for larger *Re*. We explore this in the present work. For the same we use the concept of *intermittency* which is extensively used in the description of transition from laminar to turbulent states. We denote the intermittency factor by I_f. It is the fraction of time during which the flow is in a turbulent state (Tennekes and Lumley [21]). I_f lies between 0 and 1; $I_f = 0$ implies that the flow is laminar while $I_f = 1$ means that the flow is turbulent all the time. In the present work, we utilize I_f to quantify the fraction of time during which LSB exists in the flow.

The present work aims at providing further insight into the dynamics of LSB on circular cylinder. A stabilized finite element is utilized to carry out three-dimensional computations. The governing equations are the incompressible Navier-Stokes equations in the primitive variable. Computations are carried out for $1 \times 10^4 \leq Re \leq 4 \times 10^5$. The time histories of the force coefficients as well as the surface pressure, along with the other flow variables, are analyzed to investigate the LSB. The paper is organized as follows. Section 2 gives the details of the finite element mesh and the numerical method used for the computations. The phenomena of drag crisis along with other characteristics of the flow observed in various regimes is discussed in Section 3. The intermittent nature of LSB is also explored. Finally, concluding remarks are made in Section 4.

2. Numerical method and computational details

2.1. The finite element formulation

The flow is modeled by the incompressible Navier-Stokes equations in the primitive variables. A stabilized finite element formulation (Tezduyar et al. [22]) is utilized to discretize the equations. The second-order-accurate-in-time, Crank-Nicholson scheme is employed for time integration. The Galerkin formulation is unstable for advection dominated flows as the cell *Re* increases beyond a certain value. It also constrains the combination of interpolation functions that can be utilized for velocity and pressure. To circumvent these numerical instabilities, a stabilized formulation is used. The Streamline-Upwind/Petrov-Galerkin (SUPG) formulation for solving convection dominated flows was introduced by Brooks and Hughes [6]. Hughes and Tezduyar [10] introduced the SUPG method for flows governed by the compressible Euler equations. Hughes et al. [9] proposed a procedure to stabilize the computations for Stokes flows even with velocity-pressure elements that do not satisfy the Brezzi condition. It enables the use of equal-order interpolations for velocity and pressure without producing spurious numerical oscillations in the pressure field. This type of stabilization (Pressure-Stabilizing/Petrov-Galerkin (PSPG)) was generalized to finite Reynolds number by [22]. In this work the streamline-upwind/Petrov-Galerkin (SUPG) and pressure-stabilizing/Petrov-Galerkin (PSPG) method is utilized (Tezduyar et al. [22]) to stabilize the computations. The linear algebraic equations resulting from the finite element discretization are solved using a matrix-free implementation of the Generalized Minimal RESidual (GMRES) technique (Saad and Schultz [16]) in conjunction with diagonal preconditioners. To handle large scale computations, the solution method is implemented on a distributed memory parallel computing machine. For more details on the finite element formulation and its parallel implementation the reader is referred to the article by Behara and Mittal [4]. The method has been successfully applied to solve various flow problems in the past (Singh and Mittal [19] and Tezduyar et al. [22]).

Computations are carried out in both two- and three-dimensions. Detailed results for the 2D computations were presented in our earlier study (Singh and Mittal [19]). In this work, we focus on 3D computations; the results for 2D computations are presented mostly for comparison, and to bring out the three-dimensional effects. At the *Re* considered in this study, the flow involves small scale flow structures. It is not possible to resolve the flow at all the scales via a Direct Numerical Simulation (DNS), with the present computational resources, as the number of grid points required is extremely large. We present two sets of computations. In the first set, the smaller unresolved flow structures are modeled using a sub-grid scale model. Large Eddy Simulation (LES) is employed; the effect of small scales is represented by a constant coefficient ($C_s = 0.1$) Smagorinsky model. The implementation of the model is similar to that used by Johari and Stein [11]. In the Smagorinsky model, an eddy viscosity is added to the molecular viscosity. It is given as $\mu_t = \rho(C_s h_e)^2\sqrt{2\epsilon(\mathbf{u}) : \epsilon(\mathbf{u})}$, where h_e is the element length scale, and $\epsilon(\mathbf{u})$ is the symmetric part of the velocity gradient. The second set of computations, in this work, are 'model free'. The numerical viscosity introduced by the stabilization terms drains out the energy that the eddy viscosity is expected to do for the unresolved small scales. Singh and Mittal [19] and Akin et al. [2] showed that in two dimensions, with linear elements, the numerical diffusion due to the stabilizations is much higher than the eddy viscosity generated by the Smagorinsky model except in regions very close to the cylinder where it