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Numerical analysis of laminar and transitional flow in a planar sudden expansion

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ABSTRACT

The laminar and transitional flow through a planar sudden expansion with various expansion ratios is studied using two-dimensional direct numerical simulations at flow Reynolds numbers up to 5000 and expansion ratios in the range of 1.33 to 4.00. Results show the relationship between the reattachment length and the flow Reynolds number and expansion ratio. Correlations are developed for the non-dimensional reattachment length and the maximum velocity magnitude in the reverse-flow region in terms of Reynolds numbers and expansion ratios. The correlations and computed results achieve excellent agreement with published literature. Bifurcation phenomena resulting in the loss of flow symmetry downstream of the sudden expansion is observed and critical Reynolds numbers for the onset of bifurcation for various expansion ratios are identified. The initiation mechanism for bifurcation is attributed to the amplification of streamwise velocity disturbances at a spatial location that depends on the flow Reynolds number. For Reynolds numbers near the critical value for bifurcation onset, the disturbances grow from the sudden expansion, while for Reynolds numbers well above the critical value, the disturbances grow from the location of flow reattachment that occurred in the symmetric condition prior to bifurcation. Finally, the effect of the bifurcation phenomena on the development of flow unsteadiness and transition to turbulence downstream of the sudden expansion is linked to the successive formation of smaller-scale, steady recirculation zones. Transition is initiated by inviscid instability of the inflectional velocity profiles associated with these smaller-scale zones.

1. Introduction

The flow of an incompressible fluid in a sudden planar expansion is representative of numerous flows in pipeline networks, combustion chambers, and fluidized beds [1–4]. The heat-transfer coefficient and pressure drop through a sudden expansion are strongly affected by the length of a reversed-flow region that forms immediately downstream of the expansion due to the high momentum exchange that occurs in that region [5]. This momentum exchange improves mixing of the flow, and so applications requiring mixing of the flow may benefit from the addition of a sudden expansion to promote passive mixing. For instance, Forrest et al. [6] employed a sequence of sudden expansions in a fluidized bed reactor to enhance the mixing of reagents in a reactor designed for removing crystalline phosphorus from wastewater streams. Considering the importance of the reversed-flow region, design optimization of engineering devices in which sudden expansions are used to promote passive mixing requires empirical correlations that can accurately predict the reattachment length and the magnitude of flow reversal for a range of Reynolds numbers and geometrical configurations.

Table 1 lists the numerous experimental and computational studies that have been conducted of sudden expansions in planar and axisymmetric configurations. The primary parameters affecting sudden expansions are the flow Reynolds number based on the inlet channel height and maximum inlet velocity (Re) and the ratio of the channel height upstream of the sudden expansion to the height downstream, termed the expansion ratio (Er). The streamwise length of the reversed-flow region downstream of the expansion, termed the reattachment length, is plotted in Fig. 1 normalized by the step height (L_r/h). Several studies [7,8] have found that the streamwise length of the reversed-flow region, termed the reattachment length and denoted by L_r, increases linearly with Reynolds number up to approximately $Re = 250$, which corresponds to a laminar flow regime, and then grows exponentially in the range $400 < Re < 1000$ as unsteady fluctuations develop within the separated shear layers that occur downstream of the sudden expansion. The maximum reattachment length occurs at about $Re = 1000$, for which transition to turbulence is completed

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Table 1
Studies of shear-layer reattachment in sudden expansion flows.

Reference	*Er*	Inlet flow condition	*Re* range	Type	Geometry
Macagno & Hung [8]	2.0	Fully developed	1- 400	Experiment	Axisymmetric
Back & Roschke [14]	2.6	Uniform	40- 8400	Experiment	Axisymmetric
Iribarne et al. [15]	2.0	Uniform	180- 2710	Experiment	Axisymmetric
Feuerstein et al. [16]	3.4	Fully developed	444- 1510	Experiment	Axisymmetric
Scott et al. [11]	1.5–4.0	Fully developed	100–400	Numerical	Axisymmetric/Planar
Latornell & Pollard [10]	2.0	Fully developed & uniform	10–3000	Experiment	Axisymmetric
Pak et al. [17]	2.0–2.6	Uniform	60–80,000	Experiment	Axisymmetric
Badekas & Knight [12]	1.5–6.0	Fully developed	100–400	Numerical	Axisymmetric
Hammad et al. [7]	2.0	Fully developed	40–422	Experiment	Axisymmetric
Furuichi et al. [18]	1.8	Fully developed	500–15,000	Experiment	Axisymmetric
Dağtekin & Ünsal [13]	1.5–10.0	Fully developed	100–1000	Numerical	Axisymmetric/Planar

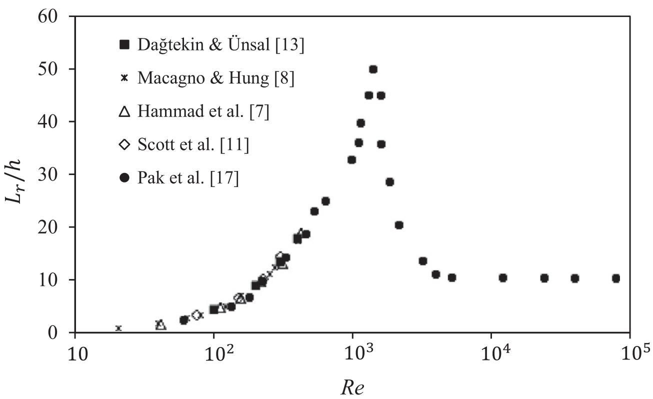

Fig. 1. Published results of shear layer reattachment length versus flow Reynolds number for $Er = 2.00$.

Table 2
Studies on bifurcation.

Reference	*Er*	$Re_{cr,\,b}$
Alleborn et al. [28]	2.0	218
	3.0	80
	5.0	42.5
Battaglia et al. [29]	1.5	446
	2.0	215
	3.0	81
	4.0	54
Drikakis [22]	2.0	216
	3.0	80
	4.0	53
Durst et al. [30]	2.0	125–200
Fearn et al. [31]	3.0	80.9
Hawa & Rusak [32]	3.0	80.7
Kadja & Touzopoulos [33]	2.0	200
Kudela [34]	3.0	84–187
Luo [35]	3.0	92.4
Manica & De Bortoli [36]	3.0	80–100
Shapira et al. [37]	2.0	215
	3.0	82.6
Schreck and Schäfer [38]	3.0	81
Neofytou & Drikakis [39]	2.0	285
Battaglia & Papadopoulos [24]	1.6	340–345
	2.0	217

at the reattachment location [9]. An abrupt decrease in the reattachment length is observed for $1000 < Re < 8000$ as the turbulent region moves upstream towards the step. Finally, a gradual increase in reattachment length is observed for $Re > 10{,}000$ as transition to turbulence occurs upstream of the expansion. Latornell and Pollard [10] note that the reattachment lengths in Fig. 1 are also sensitive to the shape of the velocity profile at the inlet to the channel; larger reattachment lengths occur for fully-developed inlet profiles compared to spatially-uniform profiles with the same flow Reynolds number. The effect of the inlet velocity profile on reattachment length is especially pronounced in the transitional Reynolds number range.

In addition to Reynolds number, expansion ratio also has a strong impact on the flow development downstream of a sudden expansion [11–13]. These studies show that the reattachment length tends to increase with expansion for a given Reynolds number, with transition to turbulence occurring at successively lower Reynolds numbers as *Er* increases. Dağtekin and Ünsal [13] provided a set of semi-empirical correlations for the reattachment length as a function of Reynolds number and expansion ratio for $Er = 1.5$–6.0 and $Re < 500$.

As the Reynolds number increases in sudden-expansion flows, numerous studies have observed an abrupt loss of symmetry in which the flow bifurcates and reattachment occurs at different locations on the upper and lower channel walls. Early experimental studies by Durst et al. [19], Cherdron et al. [20], and Ouwa et al. [21] showed that asymmetry of the upper and lower reattachment lengths occurs after a critical Reynolds number is reached. Drikakis [22] and others demonstrated that the value of the critical Reynolds number decreases with increasing expansion ratio. Papadoupoulos and Otugen [23] and Battaglia and Papadopoulos [24] showed that bifurcation is a two-dimensional instability and in 3D cases the channel aspect ratio can incorporated into an effective expansion ratio. A summary of the critical Reynolds numbers for the initial onset of symmetry-breaking bifurcation obtained by different researchers are presented in Table 2.

The onset of bifurcation is described in literature as instability of the symmetric base flow that amplifies disturbances that are initially present within the flow [22]. Mullin et al. [25] used high resolution magnetic resonance imaging techniques to identify the velocity disturbances that precede the onset of bifurication in an axisymetric sudden expansion. Based on their finding, the critical Reynolds number for onset bifurcation was estimated as $Re_d = 1139 \pm 10$ (based on the inlet pipe diameter and average inlet velocity) for an expansion ratio of $Er = 2$. Later, Sanmiguel-Rojas et al. [26] used global mode analysis to identify the unsteadiness characteristics of flow through an axisymmetric sudden expansion and tried to explain it with linear stability theory, reporting that linear instability of the axisymmetric state appears for $Re_d > 3273$, which is much higher than the critical Reynolds number measured by [25]. Sanmiguel-Rojas et al. [26] suggest that this discrepancy between the measured and theoretical critical Reynolds number is due the sensitivity to the disturbance levels in the incoming flow upstream of the expansion. The incoming disturbances are found to be amplified by the sudden expansion in which the larger expansion ratios tend to intensify bifurcation at the expansion step and also accelerate transition to turbulence [27]. The non-linearity in the noise amplification and transition processes has resulted in