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Non-reflecting coupling method for one-dimensional finite difference/finite volume schemes based on spectral error analysis

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ABSTRACT

For the compressible coupled simulation of piping systems including fluid machinery, an embedment of 3D finite volume schemes for active components such as pumps and compressors into 1D finite difference characteristic methods for passive components (e.g. pipes, valves) offers a sophisticated 3D investigation of the machinery with a moderate simulation effort for the entire system. While a 3D-1D coupling method for finite volume Godunov schemes is available from preliminary studies, in the present paper, a new non-reflecting coupling method for 1D finite volume Godunov and 1D finite difference characteristic methods is presented, and the mechanisms for spurious reflections at the coupling interface are revealed. The proposed methodology is based on spectral error analysis and is in general applicable to the coupling of any fundamentally different numerical schemes. It is demonstrated that by matching the numerical diffusion and speed of sound in both coupled domains, reflections can be minimized.

1. Introduction

Fluid machinery strongly interact with their facility by pressure and velocity pulsations that propagate through the compressible fluid into the connected piping systems: positive displacement pumps and compressors inherently generate a periodic pulsation, and centrifugal pumps and compressors also generate a periodic disturbance e.g. due to the impeller volute tongue interaction; in cavitating flows the collapse of voids generates strong shock waves that also propagate into the connected pipe system and may cause reflections. The geometry of piping systems may have a major feedback to the flow and acoustic field within the machine due to reflections and superposition of propagating waves [1–6] . Thus, in order to take into account the interaction of fluid machinery with connected pipes, the consideration of a coupled fluid system of interacting passive (e.g. pipes, valves) and active (pumps, compressors) components is indispensable.

Numerical simulation of compressible unsteady flows in fluid systems and fluid machinery is an established means for the design, analysis and optimization of machinery, components and piping systems. There is a huge variety of discretisation schemes and solution methods, which are based on particular mathematical forms of the underlying governing equations e.g. weak or normal formulation which describe the conservation of mass, momentum and energy [7]. For several system components, such as pipes, elbows or tees, the assumption of plane wave propagation is generally valid [6]. Thus, as long as the dominating pulsation frequencies are below the cut-off frequency for acoustic cross-wall modes 3D simulation of these components does not provide significantly more information than 1D simulation concerning wave propagation, but creates a large amount of computational effort and redundant data. Therefore, wave propagation in pipe systems and selected components may be simply calculated by 1D simulation tools. Characteristic methods based on the normal form of the governing equations in combination with a finite difference discretisation [8–10] are convenient for fast hydraulic and pneumatic system simulation due to their computational efficiency. A further major benefit of characteristic methods is their numerical stability as well as straightforward implementation of characteristic boundary conditions with well-defined and controllable reflection properties [11,12]. Viscous flow can be modelled by a friction source term [9,13–16]. Characteristic methods are well-established for pipe network as well as hydraulic and pneumatic fluid system design in industrial processes [3,16–22]. For simulations with characteristic methods, the effect of piston movement (reciprocating machinery) or impeller - volute tongue interaction (turbo machinery) is prescribed as a pulsation source, i.e. pulsating flow and pressure field [3,17,23].

Despite the maturity and advantageous properties of 1D characteristic finite difference schemes, there are several drawbacks,

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Nomenclature

Latin letters

a	speed of sound $\left[\mathrm{m\ s^{-1}}\right]$
c	flow velocity $\left[\mathrm{m\ s^{-1}}\right]$
CFL	CFL-number $[-]$
f	frequency [Hz]
$\vec{F}_{num}$	flux $[\mathrm{kg\ m^{-2}\ s^{-1}}$ $\mathrm{Pa\ m^{-2}}]$
j	imaginary unit $[-]$
k	wave number $\left[\mathrm{m^{-1}}\right]$
L	domain length [m]
Ma	Mach number $[-]$
Ma_{ac}	acoustic Mach number $[-]$
n_{iter}	number of time steps $[-]$
p	pressure [Pa]
r	reflection coefficient $[-]$
R_a	ratio of num. speed of sound $[-]$
R_d	ratio of diffusion errors $[-]$
$\mathrm{Re}()$	Operator: Real part of a complex number $[-]$
t	time [s]
Δt	temporal discretisation [s]
T	temperature [K]
x	spatial coordinate [m]
Δx	spatial discretisation [m]
Z_s	specific sound impedance $\left[\mathrm{N\ s\ m^{-3}}\right]$

Greek letters

α	damping constant $\left[\mathrm{m^{-1}}\right]$
β	propagation constant $\left[\mathrm{m^{-1}}\right]$
ϵ_d	diffusion error $[-]$
ϵ_ϕ	dispersion error $[-]$
λ	wavelength [m]
ρ	density $[\mathrm{kgm^{-3}}]$
ϕ	numerical phase angle [°]
φ	phase angle [°]
ω	circular frequency [Hz]

Subscripts

A/B	base points
c	velocity related
eva	evaluation point
$exact$	exact
exc	excitation
i	grid point
if	interface
inc	incident
$init$	initial condition
L	left-hand
min	minimum
num	numerical
phy	physical
p	pressure related
R	right-hand
ref	reference solution
$reflect$	reflected
s	isentropic
1/2	domain number
0	inviscid
$\_$	complex number

Superscripts

n	time step
$\hat{}$	amplitude
$\bar{}$	time averaged mean flow values

Abbreviations

1D	one-dimensional
3D	three-dimensional
FD	finite difference
FV	finite volume
nrbc	non-reflective boundary condition

which make them unattractive for example for multiphase flow (e.g. cavitation, gas laden liquids), or complex 3D geometries and flows within fluid machinery. Finite difference schemes may not be strictly conservative, and an accurate representation of weak discontinuous solutions may demand the use of finite volume schemes. For an accurate resolution of pressure or even shock wave propagation, finite volume Godunov-type schemes [24–26] which are based on a strictly conservative integral formulation of the governing equations and solve local Riemann problems at the cell faces are an established choice for highly-compressible flows as high-speed or multiphase flows. Several computationally efficient approximate Riemann solvers, e.g. [27–32] are available. There are numerous particular adaptions and extensions of Godunov schemes, e.g. for multiphase and multi fluid flow [33–38], among many others. Besides occasional applications of finite volume schemes to 1D pipe flow as for example cavitating flow in hydraulic systems [14,39,40] or water network simulation [41], their main field of application is 3D simulations.

3D simulation of entire fluid systems is however concerned by an excessive computational effort and still limited by available computational resources and therefore commonly restricted to refined investigations of only few selected components of the fluid system, e.g. fluid machinery. To reduce the influence of boundary conditions, a certain amount of piping at the inlet and the outlet can be included in the 3D model. Although artificial wave reflections by boundary conditions may be reduced by non-reflecting boundary conditions [12], the compressible interaction of fluid machinery with piping systems, i.e. the travelling and interaction of waves from the 3D machinery component into the piping system and vice versa, can inherently not be taken into account by the use of boundary conditions. An appropriate approach is the fully-coupled embedment of 3D simulation models into a 1D simulation environment. Due to the above discussed particular properties of different numerical schemes, a 3D simulation by means of a Godunov finite volume scheme and 1D simulation of the connected passive components by means of characteristic finite difference methods in combination with suitable coupling algorithms is a feasible procedure.

A precondition to a feasible coupling algorithm is the conservation of mass, momentum and energy as well as undisturbed wave propagation without artificial wave reflection and generation. Such perturbations may severely disturb and deteriorate the numerical solution. Exemplary 3D-1D coupling applications on engines [42–46] and turbo machinery [47–49] are based on an interface treatment by exchange of primitive variables and mutual time-varying prescription of boundary conditions. For simulation of blood flow in arteria coupling of 3D and 1D domain is carried out by conservation of mass flux and total normal stress at the interface [50–53]. In a coupling of 3D cavitating water turbine flow with a 1D draft tube model [54] and orifice cavitation in a closed-loop channel [55,56] the fluid is assumed to be incompressible in the 3D domain, and pressure wave propagation is considered solely in the 1D domain. A coupling of a compressible 3D finite volume scheme and a 1D characteristic finite difference scheme, based on the exchange of Riemann invariants is presented by Galindo et al. [57]. These exemplarily cited studies have in common that discussions