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# Conservative high-order flux-reconstruction schemes on moving and deforming grids

Yoshiaki Abe[a,*], Takanori Haga[b], Taku Nonomura[c], Kozo Fujii[c]

[a] Department of Aeronautics and Astronautics, University of Tokyo, Sagamihara, Kanagawa, 2525210, Japan
[b] JAXA's Engineering Digital Innovation Center, JAXA, Sagamihara, Kanagawa, 2525210, Japan
[c] Institute of Space and Astronautical Science, JAXA, Sagamihara, Kanagawa, 2525210, Japan

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ABSTRACT

An appropriate procedure to construct symmetric conservative metrics is presented for the high-order conservative flux-reconstruction scheme on three-dimensionally moving and deforming grids. The present framework enables direct discretization of the strong conservation form of governing equations without any errors in the freestream preservation and global conservation properties. We demonstrate that a straightforward implementation of the symmetric conservative metrics often fails to construct metric polynomials having the same order as a solution polynomial, which severely degrades the numerical accuracy. On the other hand, the symmetric conservative metrics constructed using an appropriate procedure can preserve the freestream solution regardless of the order of shape functions. Moreover, a convecting vortex is more accurately computed on deforming grids. The global conservation property is also demonstrated numerically for the convecting vortex on deforming grids.

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## 1. Introduction

Recently, a number of methods with high-order spatial accuracy have been developed on unstructured grids, e.g., discontinuous Galerkin (DG), spectral difference (SD), spectral volume (SV), and flux reconstruction (FR) schemes. [8,10] These unstructured high-order schemes introduce multiple degrees of freedom (DoF) for achieving high-order accuracy in each computational cell (or element), and the flux at the cell boundaries is computed using an approximate Riemann solver. Based on the unstructured grids, these schemes are capable of high-order discretization of flow fields around more complicated geometries than conventional finite-difference schemes on structured grids. In addition, a high-order shape function can be applied with multiple inner grid points so that the shape of each cells is represented as a high-order curved element, i.e., a high-order mesh [1,28]. Although a research on the high-order mesh generation is still developing [28], it would show the significant potential for the smooth and fine representation of curved boundary with considerably coarse node points. Furthermore, several benchmark cases have been reported in which these unstructured high-order schemes achieve almost the same resolution as conventional high-order finite-difference schemes with comparable total DoF on the structured grids [27]. This study focuses on the FR scheme [10], which recovers standard high-order schemes such as nodal DG, SD, and SV by different choices of correction functions for linear problems. Specifically, this study investigates the conservative FR scheme [8], in which the governing equations are expressed in the strong conservation form. The strong conservation form is important for correctly computing the shock jump conditions and speed [8] and is considered to be the basis for kinetic energy preserving schemes on the curvilinear elements [7,16,20], which are formulated as a combination of discretizations of conservative and nonconservative governing equations.

When computing flows around complex geometries using conventional high-order finite-difference schemes, a body-fitted (generalized) coordinate system is frequently adopted. In this construct, the fidelity of the represented boundary shape directly depends on the number of grid points. In contrast, in the FR scheme, the boundary shape of each cell is analytically defined by a high-order shape function, i.e., high-order curved mesh. Despite the strong conservation form of the governing equation, the use of a three-dimensional body-fitted coordinate system and high-order shape functions often fails to compatibly satisfy both the freestream

* Corresponding author. Tel.: +05033624113.
*E-mail addresses:* abe@flab.isas.jaxa.jp (Y. Abe), haga.takanori@jaxa.jp (T. Haga), nonomura@flab.isas.jaxa.jp (T. Nonomura), fujii@flab.isas.jaxa.jp (K. Fujii).

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preservation [13] and global conservation properties [2,3]. The failure in the freestream preservation property arises from violation of the Leibnitz rule and the commutative property of multiple differencing operations when a coordinate-transformation matrix (i.e., metrics) is inappropriately discretized. The conditions for freestream preservation are expressed by metric identities, which are collectively referred to as the geometric conservation law (GCL). The GCL identities comprise the surface closure law (SCL) and the volume conservation law (VCL) [23]. Although the freestream preservation property on a moving and deforming grid is ensured by satisfying both the SCL and VCL identities, only the VCL identity is considered herein. This is because we demonstrated an appropriate procedure for constructing spatial metrics satisfying the SCL identities on a stationary grid in a previous study [1].

In the FR/CPR framework, Liang et al. [14] demonstrated a method for preserving the freestream by directly discretizing a nonconservative governing equation. Although the global conservation property is satisfied if the discretization of the nonconservative form is exactly equivalent to that of the strong conservation form using exact discretizations for time and spatial derivatives, the conservation property was not numerically verified in their study [14]. In practice, typical discretizations of time and spatial derivatives using a polynomial approximation often introduce truncation and aliasing errors, which leads to the numerical violation of the commutative property of the strong conservation form and the nonconservative form. On the other hand, Gao and Wang introduced a chain-rule (CR) method with a correction for the global conservation property on stationary grids [6]. Since this method adopts nonconservative governing equations, the failure in the freestream preservation property does not arise. Moreover, a computational error in the global conservation property is corrected for by adding a correction source term in each cell. The compatibility of the freestream preservation and global conservation properties might be able to be extended to moving and deforming grids. However, its implementation is not trivial, and verification is required. In addition, since the correction source term is uniformly imposed in each cell as an averaged value, a numerical solution would be different from that obtained by a direct discretization of the strong conservation form. Such a difference will be small if the flow field is smooth. However, the properties of the correction procedure have not been adequately investigated for complex flows that include discontinuities such as shock waves.

In this paper, we introduce a method for directly discretizing the strong conservation form without any errors in either the freestream preservation or global conservation properties on moving and deforming grids in the FR framework. In order to achieve the freestream preservation property, we apply a versatile technique developed in a high-order finite-difference framework [2–5,17,18,23,25,26], in which the coordinate transformation metrics are analytically re-expressed in conservative form (hereinafter referred to as *conservative metrics*). Although the use of the conservative metric is similarly expected to ensure the freestream preservation property in the FR scheme, its implementation is not as straightforward as in the previous study for spatial metrics on stationary grids, i.e., satisfying the SCL identities [1]. Notice that the present framework would also be effective for satisfying the GCL identities directly in an arbitrary Lagrangian–Eulerian (ALE) method [9]. The ALE method in a DG formulation has been developed by many researchers [15,19,24], where the GCL identities are frequently satified by another technique, e.g., solving an additional scalar equation to compensate for the discretized GCL errors [19], which would be simply replaced by the use of the present conservative-metric technique even for the high-order shape function.

In the present paper, the following items are newly discussed for the FR scheme:

1. For moving and deforming grids with the high-order shape function, compatible conditions for satisfying both the freestream preservation (i.e., the GCL identities) and global conservation properties are presented (Section 3).
2. It is shown that using a conventional formulation of coordinate transformation metrics, freestream cannot be preserved regardless of the order of solution polynomials on moving and deforming grids in the FR framework. Then, an appropriate construction method is proposed for the conservative metrics, which satisfies the freestream preservation property without neglecting the global conservation property on moving and deforming grids using an arbitrary-order shape function (Section 4).
3. The resolution and accuracy of a numerical solution are verified for the proposed conservative metric formulations on a moving and deforming grid. The computational accuracy of the scheme with an appropriate conservative-metric formulation is compared with that with a straightforward formulation of metrics (Section 5).

## 2. Conservative flux-reconstruction scheme

### 2.1. Coordinate systems

Since moving and deforming grids are considered herein, the following assumptions are introduced for a coordinate transformation between the Cartesian and body-fitted coordinate systems:

$$\xi = \xi(x, y, z, t), \quad \eta = \eta(x, y, z, t), \quad \zeta = \zeta(x, y, z, t), \tag{1}$$

$$x = x(\xi, \eta, \zeta, \tau), \quad y = y(\xi, \eta, \zeta, \tau), \quad z = z(\xi, \eta, \zeta, \tau), \tag{2}$$

$$t = \tau, \tag{3}$$

where $t$ and $\tau$ are the physical and computational time, respectively. The computational domain is spatially subdivided into hexahedral cells at each time step. Each cell in the Cartesian coordinate system $\{x, y, z, t\}$ of physical space is mapped onto a standard cube cell $E_s := \{\xi, \eta, \zeta, \tau \mid -1 \le \xi, \eta, \zeta \le 1, \tau_{l_{\min}} \le \tau \le \tau_{l_{\max}}\}$ in the body-fitted coordinate system $\{\xi, \eta, \zeta, \tau\}$ of computational space. Here, $\tau_{l_{\min}}$ and $\tau_{l_{\max}}$ indicate lower and upper bounds of the time steps, respectively, which are used for time integration to update a solution at $\tau = \tau_l$.

The shape of the $n$th cell, i.e., $\boldsymbol{r}_n(\xi, \eta, \zeta, \tau) := x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$, is approximated by a tensor product of $N$th- and $T$th-order polynomials in the spatial and temporal directions, respectively. Therefore, grid points in the $n$th cell are defined for $(N+1)^3$ in the spatial direction and $(T+1)$ in the temporal direction. Hereinafter, the grid point is referred to as *GP*, which is used to indicate both the node and inner grid points. Fig. 1 shows GPs as blue points when $N = 1$ and $T = 1$ are assumed. When $N \ge 2$, a GP should be defined not only at the cell vertex but also at the cell boundary and interior of the cell. In this study, we assume that all GPs are provided by the computational grid file and the augmented high-order GP is defined analytically (see Eqs. (63)–(65) in Section 5). On the other hand, the degree of the temporal polynomial is equivalent to the order of a time integration method for the solution. The reason for this is that the FR scheme generally adopts a conventional time-integration method, e.g., the two-stage Runge–Kutta (RK) method. In this case, the solution points (hereinafter referred to as *SP*: green points in Fig. 1) and GPs are collocated in the time direction. Note that SP and GP can be independently defined such that polynomials of different degree are adopted for time evolution of the grid and solution, e.g., space-time formulation [11], which, for the purpose of a concise implementation, will not be discussed herein.

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