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A statistical method for the evaluation of projectile dispersion

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ABSTRACT

As part of a research program, an extensive study on the dispersion characteristics of eight different 7.62 × 51 mm ammunition types was conducted. The paper presents the main steps in the experimental and analytical process carried out to evaluate, namely to measure and compare, the dispersion characteristics of the ammunitions; namely, (1) identify the number of rounds to fire in the trials, (2) establish a test plan and the setup for the precision trials, (3) fire the rounds, following an established protocol for the experiments, (4) collect the impact points, and measure the performance through statistical measures, (5) perform a statistical analysis of dispersion applied to the results obtained in the trials, and (6) conclude on the ammunition characteristics. In particular, the paper proposes a statistical method to evaluate the precision of ammunitions fired with precision (Mann) barrels. The practical method relies on comparison of confidence intervals and hypothesis testing on the standard deviation of samples, namely the impact points. An algorithm is proposed to compare the variances of two or more populations of ammunitions.

1. Introduction

The paper presents the main steps of the experimental and analytical process carried out to evaluate the dispersion characteristics of eight different 7.62 × 51 mm ammunitions. Briefly, through repeated firings of a number of rounds, complying with a uniform *modus operandi*, and by means of a post-trials statistical evaluation of the collected impact points, one expects to be able to distinguish among the precision obtained with the various ammunition types. However, several questions arise. To what extent can a distinction be made? Can one establish, with a limited number of rounds, that one ammunition type is superior to another in terms of its statistics of dispersion? Is the difference in performance between two different ammunitions simply due to the limited number of rounds fired? What is the confidence level in the evaluation of the dispersion? Very few answers can be found in the literature. These questions are addressed in the paper.

There are eight 7.62 × 51 ammunition types to test. The ammunitions are labeled and succinctly described as follows: A1 (open tip, boat tail, 168 gr), A2 (Gilded metal jacket, 148 gr), A3 (Full metal jacket boat tail, 170 gr), A4 (Full metal jacket, 176 gr), A5 (Gilded metal, open tip, boat tail, 175 gr), A6 (open tip, boat tail, 175 gr), A7 (open tip, boat tail, 175 gr), and A8 (175 gr). The rounds are fired with three types of 508 mm long precision (Mann) barrels: barrel with a twist of 1 turn in 254 mm or 10 inches (twist of 1/10), barrel with a twist of 1 turn in 285.75 mm or 11.25 in (twist of 1/11.25), and barrel with a twist of 1 turn in 304.8 mm or 12 in (twist of 1/12). Barrel twist determines the roll rate of the gyrostabilised projectile.

The raw measure of dispersion, prior to a statistical analysis and processing of the results, is the observation of the grouping of the impact points on target. If all rounds are fired toward the same aim point, assuming that the same equipment is used for all the rounds, then the proximity, or the grouping, of the impact points obtained after repeated firings is the first observation one can make on the level of performance of given ammunitions, Mann barrels, and shooter-target ranges [1–3].

A relatively small grouping of the impact points generally indicates a relatively high level of consistency (or precision) [1]. Assuming that one fixes, and maintains constant, the firing conditions and the shooter-target range, that wind speeds are below an acceptable threshold, and that the data acquisition system is tuned uniformly for all tests, one thus has the conditions for a fair comparison. Yet, the comparison has to be made with care, using the appropriate statistical tools and methods, and with the correct interpretation of the tests, especially if the patterns of the groupings appear, at first glance, to be relatively similar for the different

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ammunitions.

A statistical evaluation is therefore necessary to infer precision characteristics for an ammunition type, and to distinguish among the characteristics of the different ammunition types. One collects measurements, and then concludes on the ammunition characteristics in the context of the trials. This is the evaluation considered in the paper. To conduct the evaluation, a statistical method for dispersion analysis is proposed. The practical method relies on comparison of confidence intervals and hypothesis testing on the standard deviation of samples, namely the impact points. A relatively simple iterative algorithm is proposed to compare the variances of two or more populations of ammunitions.

The paper is divided as follows. Section 2 presents the calculation of the number of rounds to fire in trials, the test plan and setup for the precision trials, and the actual protocol followed during the experiments. Section 3 provides the main results collected during the trials in the form of statistics, and measures of performance, on projectile impact point locations. Section 4 focuses on the statistical analysis of dispersion using the results obtained in the trials. A statistical method that relies on hypothesis testing and on well-known tests on the standard deviation of samples, and a relatively simple iterative algorithm that enables a comparison of variances for two or more populations are proposed in Section 4. Finally, conclusions on the ammunition characteristics are stated in Section 5.

2. Pre-trials calculations and preparation

Prior to carrying out the precision trials for the eight ammunition types, and for the three Mann barrel types, a preparation phase is needed. This is the topic of this section.

2.1. Number of rounds to fire

The number of rounds must be such that the measurements expected from the trials are statistically relevant; in other words, there must be enough rounds fired to carry out the evaluation. Yet, to control costs and the duration of the trials, it is clear that one wants to fire a number of rounds that is reasonable. Thus, one seeks the minimum acceptable number of rounds.

To set the number of rounds, one relies on (1) a generalized full factorial design (GFFD) method [5,6], and (2) sample size determination methods (SSDM) that seek to constrain standard deviation differences with that of the population [6–8].

In this paper, a population is the set of all impact points associated with an ammunition type, a range and a barrel type. A sample is simply a subset of the population. Trials measurements constitute a sample.

2.1.1. Mann barrel selection

There are three barrel types (twists of 1/10, 1/11.25 and 1/12), and three copies of each type of barrel. There is therefore nine barrels in total. One would like to select a single barrel among the three copies of the same type, and do so for each barrel type. To select the barrels, a number of rounds are fired with the nine barrels using only ammunition A1, at a range of 200 m, assuming that the distribution of the impact points is normal, and assuming that each firing is an independent event. Then one uses the selected barrel with twist of 1/10, that with twist of 1/11.25 and that with twist of 1/12 to carry out the remainder of the precision trials, to fire all eight ammunition types.

GFFD is run in MINITAB® [6] with a single factor (barrel type, the independent variable), three levels (barrel twists), power p set to 0.9, significance level α set to 0.05, maximum range of differences in the dependent variable, such as impact point locations, set to $d =$ 0.4 cm [4], and standard deviation (SD) of 1 cm. The latter values are selected from past experiences. Obviously, the estimated SD value may be different from the one calculated with the trials data, once the data is collected. Yet, a value must be set to serve as an approximation and giving a general idea of the number of rounds required. Furthermore, the value of 0.4 cm may be reduced, at the cost of a significant increase in the sample size required, which is to be avoided here. The resulting number of rounds to fire is 160 per precision barrel copy. With this number of rounds, one may detect differences of 0.4 cm in the main effects with a power value of 0.9, for three copies of barrels of the same type.

Power (typically close to 1) is the likelihood that a significant difference (or effect) is identified when one truly exists [5]. The significance level (typically small) can be interpreted as the risk of obtaining false differences in the results, on the hypothesis [4,5].

The SSDM of [7,8], establish the number of rounds to fire such that one may bound the difference between the calculated SD of the sample with that of the population to a desired value. There is typically a level of confidence associated with the bound. Here, one sets a 95% confidence level. The SSDM indicate that with 160 rounds, one obtains a SD of the sample within between 10% and 15% of the SD of the population. It should be noted that the contour plots of [8] as well as the other SSDM exhibit a sharp increase in the sample size if one requires errors below 10% in SD (sample versus population statistics).

2.1.2. Evaluation of precision

There are two factors of influence: barrel type and ammunition type. There are 8 levels for the ammunition type factor. There are 3 levels for the barrel type factor. Recall that there is a single copy of a barrel type and there are three barrel types (twist), following the barrel selection process. The values for the key GFFD parameters are set to $d = 0.1, 0.2, 0.3, 0.32, 0.35, 0.40, 0.50$ cm, $p = 0.9$, $\alpha = 0.05$, and SD $\sigma = 1$ cm.

The results of the application of GFFD in MINITAB® [6] are shown in Fig. 1. From the figure, one may note: (1) for a fixed number of replicates (Reps), which correspond to the number of rounds per ammunition per barrel, a reduction in maximum difference is associated with a reduction in power, (2) for a fixed level of power, reducing d results in a nonlinear increase in the number of replicates. One may select a value of 120 rounds per ammunition per barrel for an ammunition precision evaluation with maximum detection of main effects of 0.32 cm. No formal rules are used in this selection.

With the SSDM of [7,8], one obtains an error between 10% and 15% between the population SD and that calculated with a sample of 120 rounds for a given ammunition type and precision barrel assuming (1) normal distribution of the impact points, (2) and a planned value of 1 cm for the SD.

2.2. Test plan

Two in-service ammunitions, A1 and A2, are tested in part 1. Part 2 pertains to trials with the six candidate ammunitions, A3 to A8.

2.2.1. Part 1 of trials

2.2.1.1. Target and range. A target is setup indoors at a distance of 200 m from the muzzle of the barrel. A target is setup outdoors at a distance of 800 m from the muzzle of the barrel. A video camera is positioned in proximity of each target, pointing at the target, in order to identify and collect the impact points associated with each projectile.

2.2.1.2. Ammunitions and barrels. For the part of the trials pertaining to the precision barrel selection, 160 rounds are fired with