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Improved theory of projectile trajectory reference heights as characteristics of meteo-ballistic sensitivity functions

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ABSTRACT

Projectile trajectories calculated under non-standard conditions are considered to be perturbed. The tools utilized for the analysis of perturbed trajectories are weighting factor functions (WFFs) which are a special kind of sensitivity functions. WFFs are used for calculation of meteo ballistic elements μ_B (ballistic wind w_B, virtual temperature τ_B, pressure p_B, density ρ_B, speed of sound a) as well. An effect of weapon system parameters can be incorporated into calculations through the reference height of trajectory - RHT. RHT are also calculated from WFFs. Methods based on RHT are far more effective than traditional methods that use weighting factors q.

We have found that the existing theory of RHT has several shortcomings due to we created an improved theory of generalized RHT which represent a special sensitivity parameters of dynamical systems. Using this theory will improve methods for designing firing tables, fire control systems algorithms, and meteo message generation algorithms.

1. Introduction

This contribution follows up on our earlier publications [1–5], but a detailed analysis of problems under consideration is presented in studies [6–8]. For the sake of understanding of contents of this article, it is, at least, needful to peruse problems of weighting (factor) functions (curves) WFFs presented in Ref. [4]. The traditional theory of the reference height of a trajectory RHT is elaborated in the article [1] predominatingly under utilization of the textbook [10].

We apologize for erroneously referred to signs in the article [4] in the relations (33–35). In the present article they correspond to the relations (2–4), in which the signs are placed correctly. Moreover, in Ref. [4] in the relation (45): before the square bracket has to be the right number −1.

1.1. Motivation

We continue in research with the same theme and therefore our motivation can not change [1–5]: It follows from the analysis of artillery fire errors, e.g. Refs. [9,10], that approximately two-thirds of the inaccuracy of indirect artillery fire is caused by inaccuracies in the determination of meteo parameters included in the error budget model [9]. Consequently, it is always important to pay close attention to the problems of including the actual meteo parameters in ballistic calculations [10]. The following meteo parameters μ are primarily utilized: Wind vector $\boldsymbol{w}$, air pressure p, virtual temperature τ, density ρ, and speed of sound a [10–13].

This paper deals only with problems relating to unguided projectiles without propulsion system for the sake of lucidity of the solved problems.

List of notation	
μ	met parameter (element)
$\mu(y)$	real or measured magnitude of met parameter μ in height y
$r(\mu)$	weighting factor function (curve, WFF)
Q_P, Q_{CP}	effect function
$\mu_{STD}(h)$	met parameter standard course with the height h
$\Delta\mu(y)$	absolute deviation of met element μ in height y
$\delta\mu(y)$	relative deviation of met element μ in height y
$\Delta\mu_B$	absolute ballistic deviation of ballistic element μ_B
$\delta\mu_B$	relative ballistic deviation of ballistic element μ_B

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1.2. References and terminology

The reference height RHT is only in use by the Soviet methodology, which is joined with utilization of the meteo message METEO-11 ("Meteo-average"), and therefore references to this area are totally missing in publications which were published outside the former Soviet bloc. It is a problem that only the textbook [10] can be considered as quality. The next publications [14–20] give only a slight information which can serve to the understanding of basic properties of the reference height RHT and methods of its utilization. Only the article [20] deals with a new method for computation of the reference height RHT.

The new methodology was introduced (see Ref. [20]) by the instruction: "Наставление артиллерии советской армии, Артиллерийская метеорологическая служба, Москва 1956" ("Artillery Instruction of the Soviet Army, Artillery Meteorological Service, Moscow 1956"). Referred material is for us yet unavailable. The methodology was gradually introduced even in the 1960s and 1970s in the Soviet satellites. In the Soviet literature there is used the original designation for the RHT "высота входа в бюллетень "метеосредний"" ("input height into bulletin "meteo-average"") or the short one "условная высота" ("conventional height"). The first designation is too long and the second one does not correspond to the physical nature of this quantity, therefore we decided to use other designations. The author of the articles [19,20] uses the sign "conditional height", which is also not too appropriate.

1.3. The main objectives of the contribution

We have shown in articles [1–3] that the Soviet methodology for the use of the reference height RHT and the meteo message METEO-11 ("meteo-average") has a greater potential for increasing the accuracy of the calculation of the corrections on the influence of meteo parameters than the traditional methodology [21–26], which was introduced already in the 1910s and 1920s and which uses a system of weights $q(\mu)$ (weighting factors – WFs) derived from the relevant WFFs. The formation and use of the meteo message METBKQ [27] is based on this methodology and also the corresponding creation and use of the tabular firing tables – TFT in practice [28–33]. The Soviet methodology used until 1956 was virtually identical, too.

If the METEO-11 is in use, then the weighting factors $q(\mu)$ are proportional only to the relative height of the individual zones – layers. The effect of ballistic parameters of weapons and ammunition is then included in the calculations just by using the reference height of projectile trajectory – RHT Y_R, which is a multiple of the height of trajectory (HT, $Y = Y_S$), thus $Y_R = K_R \cdot Y_S$, K_R = cc. 0.8 to 1.4. The values of RHT are together with HT given in the tabular firing tables for the relevant projectile and the charge depending on the range of fire. The reference heights RHT are calculated also from the relevant WFF.

In the meteo message METBKQ, the ballistics elements μ_B are sought for the height of trajectory HT Y_S corresponding to the given range X, whereas in the METEO-11, the ballistics elements μ_B are sought for the reference height RHT Y_R corresponding also to the given range X.

We have shown in the article [4] that from the view of the modern theory of sensitivity analysis of dynamical systems, the WFFs are normed sensitivity functions, which provide concentrated information about the sensitivity of changes of projectiles trajectories on the perturbations of the relevant meteo parameters μ.

The weighting factors and the reference heights RHT are derived from the WFFs and therefore they also pertain to the sensitivity characteristics of the meteo-ballistic system.

In the available literature, there exists a semi-empirical definition of the reference height RHT [10,19]. We analyzed this problem in Refs. [1–3,6]. An exact definition has not yet been published.

The main contribution of the article is an exact derivation of the definition relationship for the reference height RHT – the relation (40). This relation defines the generalized RHT [7,8], whose special case is the traditional RHT.

In practice, three perturbation models are used for the calculation of the WFFs [26,4]. The soviet methodology uses a model formulated by Frenchman P. Langevin [4,10,26]. In this case, the WFFs are without the norm effect or with a weak norm effect, therefore, the authors of the original theory of the reference height of trajectory (RHT) at all did not deal with problems of the norm effect.

Our goal was to modify the theory of RHT so as to be applicable even for perturbation model for the calculation of the WFFs used initially in the USA and now also in NATO [23,25,26]. It is typical for this model that some WFFs are with the strong and exact norm effect – paragraph 1.4.2. In this case, division by zero or by very small numbers occur in the calculations.

The practical consequence is that the calculation of respectively corrections and gradients for the numerical iteration in a neighbourhood of a strong norm effect and the exact one, either completely fails, or is very inaccurate.

Our contribution is that we can control the whole process of numerical calculations so that the needed accuracy should be achieved also in areas with strong and exact effects. Whereas, it is given "only" by the fact, that as a parameter, in all formulas, there is performing consistently the value $r(1)$ as a measure of the intensity of the norm effect.

Subsequently, we analyze three ways of numerical calculation of the generalized reference height RHT:

- our original procedure based on the definition relationship (40) – the paragraphs 2.2 and 2.3,
- the original procedure modified by us [6–8,10] – the paragraph 3.3. and relationship (65),
- the procedure proposed by Petrovic in Ref. [20], which we also generalized [8] – the paragraph 4.1. and relationship (72) and the paragraph 4.2. and relationship (78).

The published results allow generalizing of the original soviet methodology and widely putting into practice.

1.4. Weighting functions - basic information

In this paragraph, we briefly present findings from our article [4], which are necessary for the understanding of the subsequently presented theory of the reference heights RHT.

1.4.1. Green's functions of the projectile trajectory sensitivity models

The best way to analyze the characteristics of the projectiles trajectories under nonstandard conditions is the build of any of the explicit sensitivity models of projectile trajectory [21,23,25,26]. The talk is then about the (differential) sensitivity analysis of dynamical system (projectile trajectory) or about the parameter sensitivity analysis or about the sensitivity of a system to parameter variations.

The perturbation theory is often used to the build of these models [34,35]. These are the linearized models represented by systems of linear differential equations with variable coefficients.

Relations between generalized inputs (control input variables, disturbance input variables and variable parameters of the system) $z_m(t)$, $m = 1, 2, \ldots$ on one side and output variables y_l, $l = 1, 2, \ldots$ on the other hand, are given traditionally by transfer functions and Green's functions $g_{m,l}(t{-}t_P)$. There is also a generalized theory of