Energy Conversion and Management 127 (2016) 443–460

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Lambert *W*-function based exact representation for double diode model of solar cells: Comparison on fitness and parameter extraction

Xiankun Gao, Yan Cui *, Jianjun Hu, Guangyin Xu, Yongchang Yu

Key Laboratory of New Materials and Facilities for Rural Renewable Energy, Ministry of Agriculture, Henan Agricultural University, Zhengzhou 450002, China

ARTICLE INFO

Article history:
Received 24 June 2016
Received in revised form 23 August 2016
Accepted 2 September 2016

Keywords:
Solar cell
Double diode model
Lambert *W*-function
Parameter extraction

ABSTRACT

Accurate modeling and parameter extraction of solar cells play an important role in the simulation and optimization of PV systems. This paper presents a Lambert *W*-function based exact representation (LBER) for traditional double diode model (DDM) of solar cells, and then compares their fitness and parameter extraction performance. Unlike existing works, the proposed LBER is rigorously derived from DDM, and in LBER the coefficients of Lambert *W*-function are not extra parameters to be extracted or arbitrary scalars but the vectors of terminal voltage and current of solar cells. The fitness difference between LBER and DDM is objectively validated by the reported parameter values and experimental *I–V* data of a solar cell and four solar modules from different technologies. The comparison results indicate that under the same parameter values, the proposed LBER can better represent the *I–V* and *P–V* characteristics of solar cells and provide a closer representation to actual maximum power points of all module types. Two different algorithms are used to compare the parameter extraction performance of LBER and DDM. One is our restart-based bound constrained Nelder-Mead (rbcNM) algorithm implemented in Matlab, and the other is the reported R_{cr}-IJADE algorithm executed in Visual Studio. The comparison results reveal that, the parameter values extracted from LBER using two algorithms are always more accurate and robust than those from DDM despite more time consuming. As an improved version of DDM, the proposed LBER is quite promising for PV simulation and thus deserves serious attention.

1. Introduction

Ever since solar cell came on the scene, accurate modeling and parameter extraction of its nonlinear *I–V* (current vs. voltage) characteristics have drawn considerable attention as a useful tool for further simulation, evaluation, control and maximum energy harvesting of photovoltaic (PV) systems. Despite numerous models have been developed during the past decades to simulate the behavior of solar cells, only two lumped parameter equivalent circuit models are used practically: single diode model (SDM) and double diode model (DDM) [1–3]. In the equivalent circuit of DDM illustrated by Fig. 1(a), the solar cell under illumination is modeled as a photocurrent source connected with two exponential-type ideal diodes and two parasitic resistors. Diode D_1 simulates the diffusion process of the minority carriers into the depletion layer, while D_2 represents the carrier recombination in the space charge region of the junction [4]. Correspondingly, I_{D1} and I_{D2} stand for diffusion and recombination current components respectively, which are usually expressed by Shockley equation. As depicted in Fig. 1(b), SDM is developed by combining both diode currents together with the introduction of a non-physical diode ideality factor. From this point of view, SDM is a simplified version of DDM.

For a given irradiance and temperature, the *I–V* relationship in Fig. 1(a) and (b) can be represented respectively by the following DDM Eq. (1) and SDM Eq. (2).

$$I = I_{ph} - I_{01}\left[\exp\left(\frac{V+IR_s}{n_1V_{th}}\right)-1\right] - I_{02}\left[\exp\left(\frac{V+IR_s}{n_2V_{th}}\right)-1\right] - \frac{V+IR_s}{R_{sh}} \tag{1}$$

$$I = I_{ph} - I_0\left[\exp\left(\frac{V+IR_s}{nV_{th}}\right)-1\right] - \frac{V+IR_s}{R_{sh}} \tag{2}$$

where I, V, I_{ph}, I_{01}, I_{02}, I_0, n_1, n_2, n, R_s, and R_{sh} are the terminal current, terminal voltage, photocurrent, diode saturation currents, diode ideality factors, series resistance, and shunt resistance, respectively. Thermal voltage $V_{th} = N_skT/q$, where N_s is the number of cells in ser-

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* Corresponding author.
E-mail addresses: gaoxiankun78@163.com (X. Gao), cuiyan6198@163.com (Y. Cui), hujianjun@163.com (J. Hu), xgy4175@126.com (G. Xu), hnyych@163.com (Y. Yu).

http://dx.doi.org/10.1016/j.enconman.2016.09.005
0196-8904/

Nomenclature

bcNM	bound constrained Nelder-Mead algorithm
ACE	absolute current error (A)
ACE_{cal}	absolute current error of calculated current (A)
ACE_{sim}	absolute current error of simulated current (A)
DDM	double diode model
EESDM	exact explicit single diode model
fval	RMSE_{cal} obtained by the Sth run of bcNM
$f_M(V, I, X)$	error function
G	irradiance
I	terminal current (A)
I_0, I_{01}, I_{02}	diode reverse saturation currents (A)
$I_{0\min}$, $I_{0\max}$	lower and upper bounds on $I_{01,2}$ (A)
I_{cal}	calculated current (A)
I_D, I_{D1}, I_{D2}	diode currents (A)
I_{ph}	photocurrent (A)
I_{sc}	short-circuit current (A)
I_{sim}	simulated current (A)
k	Boltzmann constant ($1.3806503 \times 10^{-23}$ J/K)
LB	lower bound on $\boldsymbol{X}$
LBER	Lambert W-function based exact representation
m	parameter dimension
Max_NFEs	maximum number of function evaluations
MaxIter	maximum number of iterations
MaxFunEvals	maximum number of function evaluations
MPP	maximum power point
n, n_1, n_2	diode ideality factors
N	number of the experimental I–V data
N_s	number of cells in series
NM	Nelder-Mead algorithm
ObjFun	objective function
plotFcns	plot function
q	electronic charge (1.60217646 × 10−19 C)
r	ratio of diffusion current to the sum ofdiffusion and recombination currents
r_i	ith element of r
R_s	series resistance (Ω)
R_{sh}	shunt resistance (Ω)
RMSE_{cal}	root mean square error of calculated current
RMSE_{sim}	root mean square error of simulated current
S	restarting number of bcNM
SDM	single diode model
T	cell temperature (K)
TolFun	termination tolerance on RMSEcal (X)
TolFun_runs	RMSEcal difference
TolX	termination tolerance on X
UB	upper bound on X
V	terminal voltage (V)
V_{oc}	open-circuit voltage (V)
V_{th}	thermal voltage (V)
W_0	principal branch of Lambert W-function
X	parameter vector
X_0	initial value of X
μ	population size

ies, k is the Boltzmann constant, q is the electronic charge, and T is the absolute temperature in Kelvin and can be calculated by 273.15 plus the cell temperature in Celsius.

As can be seen from Eqs. (1) and (2), there are seven parameters (I_{ph}, I_{01}, I_{02}, n_1, n_2, R_s and R_{sh}) in DDM and five parameters (I_{ph}, I_0, n, R_s and R_{sh}) in SDM need to be extracted. The knowledge of these parameters is used not only to evaluate the performance and improve the design, fabrication process and quality control of solar cells, but also to extract the maximum power point (MPP) of PV array [5–9]. Hence, it is imperative to accurately extract these parameters from the experimental I–V data of solar cells. Unfortunately, both DDM Eq. (1) and SDM Eq. (2) are implicit nonlinear transcendental equations, mainly because neither the current I nor the voltage V can be explicitly expressed only by using elementary functions. This inherent implicit nature increases the complexity and difficulty not only of parameter extraction but also of simulation of PV systems [10], and thus calls for explicit expressions for DDM Eq. (1) and SDM Eq. (2) prior to their parameter extraction phase.

Thanks to Lambert W-function [11], which makes it possible for transforming implicit SDM Eq. (2) into the exact explicit single diode model (EESDM) Eq. (3) [12].

$$I = \frac{R_{sh}(I_{ph}+I_0)-V}{R_s+R_{sh}} - \frac{nV_{th}}{R_s}W_0(\alpha) \qquad (3)$$

where W_0 is the principal branch of Lambert W-function, and

$$\alpha = \frac{I_0R_sR_{sh}}{nV_{th}(R_s+R_{sh})}\exp\left[\frac{R_{sh}(R_sI_{ph}+R_sI_0+V)}{nV_{th}(R_s+R_{sh})}\right] \qquad (4)$$

The most desirable feature of EESDM Eq. (3) is that for any value of voltage V the corresponding exact value of current I can be calculated straightforwardly, which enables more accurate I–V characteristics [13–16], MPP tracking [17–19], optimum load [20–22] and efficient model parameter extraction [23–28]. A recent comparative study [29] revealed that Lambert W-function based analytical method [10] presents fewer errors in comparison to iterative method [30]. One of our previous studies [31] shown that EESDM Eq. (3) is much more accurate and reliable than SDM Eq. (2) in parameter extraction of solar cells. In general, EESDM Eq. (3) has better accuracy, applicability, and convergence than SDM Eq. (2) though the calculation speed is relatively lower [32].

Inspired by the superiority of EESDM Eq. (3), two Lambert W-function based explicit expressions have been developed in an attempt to approximate DDM Eq. (1). Authors in Ref. [33] reported an explicit double exponential model as an alternative to DDM. Unfortunately, this alternative model is only an approximation to DDM, since they are not exactly analogous for all possible arbitrary sets of parameters [33]. The validation results in Ref. [34] show that the equivalence between the alternative model and DDM

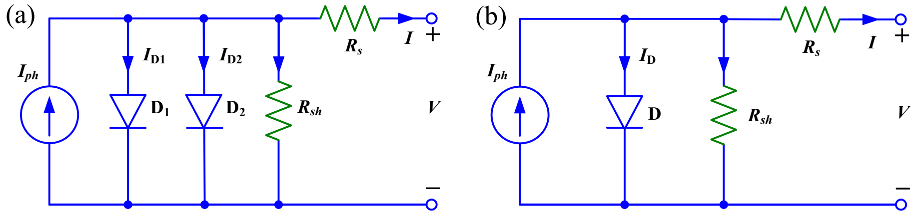

Fig. 1. Equivalent circuits of a solar cell under illumination: (a) double diode model (DDM), and (b) single diode model (SDM).