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Influences of tooth spalling or local breakage on time-varying mesh stiffness of helical gears

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ABSTRACT

Time-varying mesh stiffness (TVMS) provides important information about the health condition of geared systems. Tooth faults, like crack, pitting, spalling and breakage, will change the TVMS. This work aims to reveal the influences of tooth spalling and local breakage on the TVMS of helical gears. Firstly, an analytical model is developed to incorporate the faults by combining slicing method, discrete integral method and potential energy method. Then, parametric study is conducted. Finally, conclusions are arrived and the results indicate that length, width and location of these defects are very important parameters that need to be considered for calculating TVMS of helical gears. This study provides a theoretical basis for fault diagnosis of helical geared transmissions.

1. Introduction

Compared with spur gears, helical gears present lower noise and more steady performance and are widely employed in many kinds of rotating machineries to transmit motion and power. Time-varying mesh stiffness is one of the main vibration sources of the gear systems, especially under the condition with tooth fault (e.g., spalling, local breakage and root crack).

Lots of work could be found about the TVMS calculation for spur gears. Totally speaking, the methods could be classified into four categories: Finite element method (FEM), analytical method, combination of analytical method and FEM, and experimental method [1]. Among those methods, the analytical one based on potential energy principle is the most popular for its efficiency with enough calculation accuracy. Yang and Lin [2] firstly developed an analytical model to calculate TVMS of spur gear considering Hertzian, bending and axial compressive energy and the model was further improved by Tian [3] and Wu [4] by introducing the shearing energy. Later, improved analytical models were developed by researchers [5–8]. For the TVMS of gear pair with tooth faults like root crack, spalling, breakage and pitting, Chaari et al. [9] revealed influences of spalling and breakage on mesh stiffness and dynamic response of spur gear transmission by potential energy method. Ma and Chen [10] developed dynamics model of gear system with tooth crack and spalling failures by numerical simulation. With considerations of extended tooth contact, revised fillet-foundation stiffness under double-tooth engagement region and nonlinear contact stiffness, Ma et al. [11–14] conducted thorough research on calculating TVMS of spur gears with spalling or root crack. Liang et al. [15,16] conducted vibration signal modeling of a planetary gear set for tooth crack detection and developed model for calculating TVMS of spur gears with tooth pitting. Wan et al. [17] presented an improved method to calculate TVMS of spur gear with the potential energy stored in the part between base circle and root circle considered. Saxena and Parey [18] revealed the influences of spalling and friction on TVMS of spur gears.

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Nomenclature

A_x	cross-section area
b	tooth width
d	distance between contact point and tooth root
dA_x	cross-section area of each piece
d_1, d_2	distances from root to the starting and ending points of spalling
d_h, d_v	horizontal and vertical coordinates of the lowest point of spalling
dI'_x	area moment of inertia of each piece
dk_b^s	bending stiffness of each piece with spalling
d_{oh}, d_{ov}	horizontal and vertical coordinates of the center point of spalling
dU_b^s	bending energy stored in a piece
E	Young's modulus
F	meshing force normal to tooth surface
F_a, F_b	radial and tangential components of F
f_1, f_2	auxiliary variables, denoting distances between 2nd, 3rd contact lines and the right side of plane of action
G	shear modulus
h	distance between the contact point and center line of tooth
h_b	height of local breakage
h_x, h'_x	distance between point on tooth curve, bottom edge of spalling corresponding to the section where the distance from tooth root is x and the central line of tooth
h_s	height of spalling
I_x	area moment of inertia of the section where the distance from the root is x
i'	i'^{th} piece of the spalling portion of a tooth
k	mesh stiffness of helical gear
k_a, k_a^s	axial compressive stiffness of a tooth without and with spalling
k_{ai}	i = 1,2, axial compressive stiffness, subscript i denotes driving gear and driven gear
$k_{a1,i}$	i = 1,…,n, axial compressive stiffness, subscript i denotes ith tooth-pair
k_b, k_b^s	bending stiffness of a tooth without and with spalling
k_{bi}	i = 1,2, bending stiffness, subscript i denotes driving gear and driven gear
$k_{b1,i}$	i = 1,…,n, bending stiffness, subscript i denotes ith tooth-pair
k_f	fillet-foundation stiffness of a tooth
k_{fi}	i = 1,2, fillet-foundation stiffness, subscript i denotes driving gear and driven gear
$k_{f1,i}$	i = 1,…,n, fillet-foundation stiffness, subscript i denotes ith tooth-pair
k_h	Hertzian contact stiffness of a tooth-pair
k_i^s, k_i	mesh stiffness of ith piece with and without spalling defect,
k_s, k_s^s	shearing stiffness of a tooth without and with spalling
k_{si}	i = 1,2, shearing stiffness, subscript i denotes driving gear and driven gear
$k_{s1,i}$	i = 1,…,n, shearing stiffness, subscript i denotes ith tooth-pair
$k_{spalling}$	mesh stiffness of single tooth-pair with spalling
L_{11},L_{12},L_{13}	length of segments of contact line
L^*	auxiliary variable used in calculating fillet-foundation stiffness
l_{AP}	distance between point A and P in plane of action of helical gear
l_a	actual length of contact line with spalled tooth
l_s	length of spalling
l_t	theoretical length of contact line
M^*	auxiliary variable used in calculating fillet-foundation stiffness
N	total number of pieces of contact line
N_{11}, N_{12}, N_{13},	number of pieces corresponding to contact line length L_{11},L_{12},L_{13}
n	= $ceil(\varepsilon)$, minimum integer bigger than ε.
P^*	auxiliary variable used in calculating fillet-foundation stiffness
p_{bt}	base pitch at transverse section
Q^*	auxiliary variable used in calculating fillet-foundation stiffness
r_b	radius of base circle
S_f	auxiliary variable used in calculating fillet-foundation stiffness
t_1, t_2, t_3	auxiliary variables used in calculating actual length of contact line
t_b	thickness of local breakage
t_{be}, $t_{bs.}$	ending and starting time instants that contact line passes through breakage
t_{total}	total meshing time of single tooth-pair
U	Total potential energy stored in a pair of gears
U_{a1}, U_{a2}	axial compressive energies stored in a tooth from pinion and gear
U_{b1}, U_{b2}	bending energies stored in a tooth from pinion and gear
U_{s1}, U_{s2}	shearing energies stored in a tooth from pinion and gear
u_f	auxiliary variable used in calculating fillet-foundation stiffness
v_t	traveling velocity of meshing point along transverse section
w_b	width of local breakage

Greek symbols

α_m	acting pressure angle of gear tooth for calculating fillet-foundation stiffness
α_{s1}, α_{s2}	pressure angles corresponding to spalling area
α_1	acting pressure angle of gear tooth
α'_1	auxiliary variable
α_2	half of base tooth angle
β_b	helix angle at base cylinder
ε	total contact ratio
ε_α, ε_β	transverse and axial contact ratio
ν	Poisson's ratio
θ_b	auxiliary variable, representing the angle between the hypotenuse and the transverse section of tooth
ω	angular speed of pinion
Δy	width of a piece