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Hyperbolic constitutive model to study cast iron pipes in 3-D nonlinear finite element analyses

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ABSTRACT

Cast iron (CI) pipes were widely installed as water mains and service connections in the last century and still large numbers of CI pipes remain in service. However, many CI pipes are becoming aged and severely corroded, causing frequent pipe leaks and breaks. To better understand the failure mechanism of CI pipes, this paper provides an efficient numerical approach to analyse the behaviour of CI pipes under various loading conditions. The numerical investigation was realised by implementing a hyperbolic constitutive model (a simplified nonlinear stress-strain analysis) for CI materials into finite element analysis (FEA) in ABAQUS. Three-dimensional (3-D) FEAs were carried out for a series of uniaxial tensile and compressive tests, 3-point beam bending tests and ring bending tests on various CI pipe coupons. The stress-strain characteristics and load-deflection responses obtained from these numerical examples were validated by experimental results. The numerical results obtained from the proposed method are in good agreement with the measured data, which indicates that the mechanical performance of deteriorated CI pipes can be adequately modelled using the relatively simple nonlinear constitutive model implemented in 3-D FEA. As nonlinear behaviour has proven to be intrinsic to the widely used CI pipes, it is expected that the proposed 3-D FEA modelling technique will be of importance to the evaluation of the mechanical performance of CI pipes and, possibly, other CI structures.

1. Introduction

During the past decades, buried cast iron (CI) pipelines have received much attention from water utilities around the world as a result of increasing pipe leaks, breaks, and the incurred direct and indirect economic costs of flood damage, interruption of traffic and potable water services. CI pipes, commonly serving as pressure water mains, are made of grey iron, which were cast either in a pit or centrifugally (spun). Cast iron pipes were widely used to distribute water, gas and sewage throughout the 19th and 20th centuries, and still large numbers of CI pipes remain in service. However, the performance of water networks is adversely affected by the continuingly aging of CI pipes that have increasingly deteriorated over time.

As observed in our research work on an international collaborative research project on large diameter critical pipes (www.criticalpipes.com) [1], the stress-strain nonlinearity of grey cast iron pipes under various loading conditions and test methods (such as uniaxial tensile and compressive tests, beam and/or ring bending tests) is evident. Such stress-strain nonlinearity had

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been observed and reported as early as 1890 [2], and later by many others [3–8]. Hence, it is important to consider the nonlinear stress-strain characteristics of grey cast iron to accurately determine the pipe response.

Various nonlinear stress-strain models describing the mechanical nonlinearity of grey cast iron have been developed, among which that based on a hyperbolic function has proven to be the most suitable [9–11], given that the hyperbolic parameters are easily determined from uniaxial tensile and compressive tests. Similarly, the hyperbolic characterisation of the stress-strain relationships of CI pipes can be identified using beam bending tests; many relevant studies have been conducted by [12–14]. More recently, Rajani [8] formulated a procedure to derive hyperbolic stress-strain relationships for CI using the measured load-deflection responses of simple mechanical laboratory tests. He then applied this procedure to obtain the mechanical properties (tensile and compressive strength, and Young's modulus) from uniaxial tensile tests, beam and ring bending tests conducted by Talbot [3] on coupons taken from CI pipes. In Rajani's work, parameters for the hyperbolic constitutive model were obtained through a semi-analytical approach, which is applicable to plane strain and/or plane stress conditions. A literature review suggests that simple hyperbolic constitutive model is warranted to conduct a generic 3-D numerical analysis of CI pipes as it can reflect the complex real conditions such as the pipe geometry, randomly distributed corrosion pits and multiaxial stress state where 1-D and/or 2-D analyses may not be adequate. In this regard, the finite element method (FEM) is employed in this paper to carry out a series of numerical analyses to study the mechanical nonlinear performance of CI structures and pipes under external loads. It is worth noting that FEM has been widely used to study various pipe problems and achieved great success in recent years [15–21].

The following sections are organised as follows. The second section describes the hyperbolic nonlinear model and its implementation in 3-D finite element analyses of CI pipes. In the third section, numerical simulations on a series of uniaxial, 3-point beam bending and ring bending tests are carried out using the proposed approach; some of those tests were undertaken in the newly-developed burst test facility in the Civil Engineering Laboratory at Monash University, and others were conducted earlier by Talbot [3]. Section 4 summarises the findings of the validation of CI pipe performance obtained from the implementation of the proposed 3-D FEA approach. Finally, some issues that are worth exploring further are suggested.

2. Derivation of hyperbolic constitutive model and its implementation in 3-D FEA

2.1. Uniaxial conditions

The successful use of a hyperbolic function in modelling the nonlinear stress-strain relationship can be found in Duncan and Chang [22] for geotechnical applications. Similarly, some materials such as cast iron (CI) may also be well described by a hyperbolic function [9–11]. For CI materials, a highly nonlinear stress-strain response indicating the onset of plastic (local) yielding has long been observed at very low stress or strain state in experimental tests; further loading the CI material often leads to a significantly nonlinear stress-strain response till the brittle failure [3–8,23,24]. This nonlinearity is caused by carbon in the form of graphite flakes that act as stress concentrators; this nonlinear effect is accentuated when corroded metal irons are leached away, leaving graphite acting as voids or cracks. In uniaxial conditions, the hyperbolic constitutive model reads as follows:

$$\sigma = \frac{\varepsilon}{\alpha + \beta\varepsilon} \tag{1}$$

where, σ and ε are respectively the uniaxial stress and strain, and α and β are two parameters defining the constitutive nonlinearity. In general, the Young's modulus E is a stress- and/or strain-dependent variable. For any given deformation stage, the strain-dependent form of instantaneous elastic modulus, E, is simply derived from Eq. (1), such that:

$$E = \frac{d\sigma}{d\varepsilon} = \frac{\alpha}{(\alpha + \beta\varepsilon)} \tag{2}$$

Both parameters have physical meanings: $1/\alpha$ indicates the initial gradient of stress to strain, i.e., initial Young's modulus when $\varepsilon = 0$ in Eq. (2); and $1/\beta$represents asymptotic stress when strain increases to infinity (see Eq. (1)). Where strain is a function of stress, Eq. (1) can alternatively be expressed as follows:

$$\varepsilon = \frac{\alpha\sigma}{1 - \beta\sigma} \tag{3}$$

Substituting ε to Eq. (2), the instantaneous elastic modulus, E, in stress-dependent form is simply obtained:

$$E\frac{1}{\alpha}(1 - \beta\sigma)^2 \tag{4}$$

Use Hooke's law in incremental form to relate the stress and elastic strains.

$$d\sigma = Ed\varepsilon = \frac{(1 - \beta\sigma)^2}{\alpha}d\varepsilon \tag{5}$$