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On the modelling of mixed-mode discrete fracture: Part I – Damage models

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ABSTRACT

In this manuscript, mixed-mode fracture is studied. Conversely to previous plasticity based formulations, the purpose of this work is to derive discrete damage models to simulate the evolution of fracture under both normal and shear tractions.

First, an energy based model is used. Next, deformation-based models are adopted, both with isotropic and non-isotropic damage evolution laws. Damage is usually considered as a deformation driven process. However, fracture criteria, such as crack initiation and crack evolution, are typically defined in the stress or traction space. This is why a new, more refined model is also introduced, in which damage evolution is traction-based. Several special cases are studied, such as: homotetic damage evolution, isotropic damage evolution and a general mixed-mode evolution law. Compressive tractions are also dealt with, namely under Mode-II fracture. In all cases, as a direct consequence of the damage approach, both the total/secant constitutive relation and the corresponding incremental/-tangent stiffness are derived. Some elementary numerical results are obtained and discussed.

1. Introduction

In this work, mixed-mode fracture is studied within the framework of the damage mechanics theory. Consequently, the total secant formulation can be explicitly derived, which allows for the use of non-iterative methods, such as the ones presented in Rots [35], Rots and Belletti [36], Invernizzi et al. [17], Costa et al. [12,11]. These methods are built upon the concept of damage evolution.

Continuum damage mechanics has been used since Kachanov introduced the *effective* stress concept [19]. However, it was only in later works that continuum damage mechanics was applied to quasi-brittle materials such as concrete, namely in Kachanov [20], Krajinovic and Fonseka [22], Chaboche [10], Mazars [26], Krajinovic [21], Costanzo [13], and later in Simo and Ju [40], Mazars and Pijaudier-Cabot [27], Pijaudier-Cabot and Bažant [33], Bažant and Pijaudier-Cabot [4], Mazars and Pijaudier-Cabot [28], Peerlings et al. [32], Faria et al. [14], Simone et al. [41], Fernandez and Ayala [15], Sancho et al. [39].

The *effective* stress is associated with the hypothesis of strain equivalence [23]: "the strain associated with a damaged state under the applied stress is equivalent to the strain associated with its undamaged state under the effective stress". Thus, for a given strain, the actual stress is smaller than the *effective* stress due to the degradation of the material properties which, in quasi-brittle materials, is due to initiation, coalescence and growth of micro cracking. In a simple tensile test, the

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List of symbols

σ	stress component
$\boldsymbol{\sigma}$	stress tensor
d	scalar damage variable
$\mathbf{d}$	second order damage tensor
d_n	normal damage variable component
d_s	shear damage variable component
$\mathbf{I}$	identity tensor
Ψ	Helmholtz free energy density
Y	thermodynamic force conjugate to the internal damage variable d
$\mathbf{D}_{el}$	fourth order elastic constitutive tensor
$\boldsymbol{\varepsilon}$	strain tensor
$\mathbf{w}$	displacement jump vector
w_n	normal component of the jump displacement vector
w_s	shear component of the jump displacement vector
$\mathbf{t}$	traction vector acting at the discontinuity
t_n	normal component of the traction vector
t_n	normal traction corresponding to the maximum shear strength
t_s	shear component of the traction vector
τ_0	elastic free energy per unit area at the onset of localisation
$\mathbf{D}^{el}_{\Gamma_d}$	second order elastic constitutive tensor corresponding to discontinuity Γ_d
D^{el}_{nn}	normal diagonal component of tensor $\mathbf{D}^{el}_{\Gamma_d}$
D^{el}_{ss}	shear diagonal component of tensor $\mathbf{D}^{el}_{\Gamma_d}$
f	loading surface defined in the traction space
f_1, f_2	limit surfaces defined in the traction space
f_{t0}	initial tensile strength
f_t	tensile strength
c_0	initial cohesion: shear strength under the absence of normal traction
c	cohesion
G_0	generalised fracture energy
G_F	fracture energy
G_F^{II}	fracture energy under mode-II fracture
κ	monotonic increasing function of the displacement jump components
β	scalar function which enables transition between mode-I and modeII fracture
g	loading function defined in the displacement jump space
g_n	normal damage evolution law
g_s	shear damage evolution law
ϕ	internal friction angle
ξ_n	w_n/κ_n
ξ_s	$\lvert w_s\rvert/\kappa_s$
G	c/f_t
ρ	κ_s/κ_n

reduction of the undamaged material cross sectional area, giving rise to the definition of the average stress over a representative element volume, can be evaluated by means of a scalar variable d, called the damage variable, such that:

$$\sigma = (1-d)\bar{\sigma}, \tag{1}$$

where $\bar{\sigma}$ is the *effective* stress and $d = 0$ for undamaged material and $d = 1$ for a fully damaged material. In a more general framework, the relation in Eq. (1) can be extended to [40]:

$$\boldsymbol{\sigma} = \mathbf{M} : \bar{\boldsymbol{\sigma}}, \tag{2}$$

where $\mathbf{M}$ is a fourth-order damage tensor and ":" denotes a double contraction product. In case of non-dependence of the direction of loading we get an isotropic damage setting corresponding to (1), in which $\mathbf{M} = (1-d)\mathbf{I}, \mathbf{I}$ being the identity tensor.

For quasi-brittle materials it is usually assumed that crack initiation occurs according to Rankine-type failure criterion, i.e., when the maximum principal stress reaches the tensile strength of the material f_t. Furthermore, it is also assumed that