ARTICLE IN PRESS

Engineering Fracture Mechanics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Asymptotic finite fracture mechanics solution for crack onset at elliptical holes in composite plates of finite-width

J. Felger *, N. Stein, W. Becker

Technische Universität Darmstadt, Fachgebiet Strukturmechanik, Franziska-Braun-Strabe 7, D-64287 Darmstadt, Germany

ARTICLE INFO

Article history:
Received 29 November 2016
Received in revised form 29 May 2017
Accepted 30 May 2017
Available online xxxx

Keywords:
Finite fracture mechanics
Crack initiation
Matched asymptotic expansions
Open-hole composites
Finite-width plates

ABSTRACT

The onset of cracks at elliptical holes in finite-width laminated plates under uniaxial tension loading is investigated using a finite fracture mechanics approach. A coupled stress and energy criterion is applied in order to determine the effective strength of the notched plate and the size of instantaneously initiating cracks. A semi-analytical approach based on complex potential theory in combination with a two-scale asymptotic analysis is proposed leading to explicit relations for the stress field and energy quantities due to crack formation. This allows for an efficient finite fracture mechanics solution and a significant reduction of the numerical effort compared to a full-numerical treatment. The effect of higher-order terms in the asymptotic procedure and a technique leading to an improved convergence of the expansions are discussed in detail. A thorough comparison with numerical data is performed in order to analyse the validity range of the asymptotic solution. Furthermore, the failure load predictions are compared to experimental data from literature.

1. Introduction

Stress raisers such as open-holes, flaws or notches are immanent in engineering structures and typically represent potential loci of failure. The assessment of crack onset at such stress raisers has proven to be complicated. Classical stress-based criteria as proposed by Whitney and Nuismer [45] evaluating the stresses at a certain distance away from the stress concentration as well as fracture mechanics based concepts as, for instance, by Waddoups et al. [40] assessing the crack propagation of an inherent flaw require an internal length parameter. This is a significant drawback of these approaches since the length parameter depends on the structural configuration and hence does not represent a material parameter [38].

A very elegant concept avoiding the use of an a priori empirically determined length parameter is the coupled criterion proposed by Leguillon [22]. Assuming an instantaneous formation of finite sized cracks in the framework of finite fracture mechanics (FFM), the coupled criterion predicts crack onset if a strength and an energy criterion are fulfilled simultaneously. In this way, the coupled criterion allows for predicting the finite crack size associated to crack onset and the crack initiation load requiring only the strength and toughness as fundamental material parameters. The length parameter, the finite crack size, now results from physical considerations solely. In the recent decade, the coupled criterion has proven to be a very powerful concept that has been successfully applied in order to predict brittle failure in various structural configurations exhibiting stress raisers such as delamination in laminates due to free-edge effects [19,27], adhesive joints [7,11,20,35], bolted

* Corresponding author.
E-mail address: felger@fsm.tu-darmstadt.de (J. Felger).

http://dx.doi.org/10.1016/j.engfracmech.2017.05.048
0013-7944/

Please cite this article in press as: Felger J et al. Asymptotic finite fracture mechanics solution for crack onset at elliptical holes in composite plates of finite-width. Engng Fract Mech (2017), http://dx.doi.org/10.1016/j.engfracmech.2017.05.048

Nomenclature

Symbol	Description
a	semi-major axis of the ellipse
$\Delta a, \Delta A$	newly created finite crack length/area
Δa^*	maximum point of the two-term expansion
A_{ij}	numerical scaling coefficients
b	semi-minor axis of the ellipse
E_1	Young's modulus parallel to the fibres
E_2	Young's modulus perpendicular to the fibres
G_{12}	shear modulus
$\mathcal{G}$	differential energy release rate
$\bar{\mathcal{G}}$	incremental energy release rate
$\mathcal{G}_c$	critical energy release rate
h	plate's height
$\bar{K}_i$	intensity factor
ΔK	change of kinetic energy
K_{Ic}	fracture toughness
$\underline{n}$	normal vector
r	radial coordinate
$R_{m,n}$	m,n-Padé approximant
S_{ij}	elastic compliances
t	plate's thickness
$\underline{u}_i$	displacement eigenfunction
$\underline{U}^{\Delta a}$	solution of the displacement vector on the perturbed domain
$\underline{U}^0$	solution of the displacement vector on the unperturbed domain
W	plate's width
$\underline{x}_{ct}$	crack tip vector
x_i	Cartesian coordinate with respect to the contour of the plate's hole
$\tilde{x}_i$	Cartesian coordinate with respect to the centre of the plate's hole
y_i	scaled Cartesian coordinate in the inner domain
Γ_c	potential crack surface
η	normalised maximum point of the two-term expansion
κ	finite-width correction factor
λ	aspect ratio of the ellipse
μ_1, μ_2	roots of characteristic equation
ν_{12}	Poisson's ratio
Π	total potential energy
$\Pi^{\Delta a}$	total potential energy of the crack body
Π^0	total potential energy of the uncracked body
$\Delta\Pi$	change of total potential energy
ρ	dimensionless scaled radial coordinate
$\underline{\underline{\sigma}}$	stress tensor
σ_{ij}	components of the stress tensor
σ_∞	uniaxial far-field tension loading
σ_f	far-field stress at failure
σ_c	material strength
ϕ	circumferential coordinate
$\Psi(\cdot,\cdot)$	Psi-integral
ω	opening-to-width ratio
Ω_0	unperturbed domain
$\Omega_{\Delta a}$	perturbed domain
Ω_∞	infinite inner domain
$\underline{(\cdot)}$	first-order tensor
$\underline{\underline{(\cdot)}}$	second-order tensor
$o(\cdot), O(\cdot)$	Landau symbols

joints [8], crack initiation at V-notches [6,22,33] or U-notches [1,18,25]. In addition, the coupled criterion has been employed in order to predict the open-hole strength of quasi-isotropic composite plates [4,13,28,30,31] and of isotropic plates containing an elliptical hole [43]. An overview on recent advances and applications of the coupled criterion can be found in the review by Weißgraeber et al. [44].

Please cite this article in press as: Felger J et al. Asymptotic finite fracture mechanics solution for crack onset at elliptical holes in composite plates of finite-width. Engng Fract Mech (2017), http://dx.doi.org/10.1016/j.engfracmech.2017.05.048