Engineering Fracture Mechanics 180 (2017) 87–104

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Interactions of multiple cracks in a transversely isotropic piezoelectric plane under mixed mode condition

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ARTICLE INFO

Article history:
Received 27 February 2017
Received in revised form 20 May 2017
Accepted 22 May 2017
Available online 24 May 2017

Keywords:
In-plane
Piezoelectric material
Singular integral equations
Multiple cracks
Field intensity factors

ABSTRACT

In this paper, the interactions of multiple cracks in a piezoelectric material under the action of in-plane electromechanical loads are considered. The analytical solution of electric and Volterra climb and glide edge dislocations in an infinite piezoelectric plane are initially obtained by using the Fourier transform. The problem is then formulated by the distributed dislocation technique into a system of singular integral equations where the unknown variables are dislocation density functions on the surfaces of the cracks. These equations are solved numerically for dislocation density functions on the cracks which are used to determine field intensity factors. The results are highly affected by the interaction between multiple cracks and various loading conditions.

1. Introduction

Piezoelectric materials belong to an important class of smart materials, with wide important potential applications in electromechanical and microelectromechanical systems, such as actuators and dampers for controlling structural vibration, sensors and actuators in adhesively bonded joints, and sensors in non-destructive testing due to the intrinsic coupling characteristics between their electric and mechanical fields. The assessment of defects like micro-cracks and voids play an important role for the strength and reliability of piezoelectric components under combined electrical and mechanical loading [1]. Also, piezoelectric materials are usually brittle and contain many cracks. The interactions between the cracks may greatly affect the fracture behavior of these materials. Therefore, the study of crack problems in a piezoelectric ceramic has received much attention among the researches in recent years. Due to complexity of analyzing the cracks with exact electric boundary conditions, permeable and impermeable crack boundary conditions are commonly used in piezoelectric materials. The first type of electrical condition was suggested by Parton [2] and called an electrically permeable crack. Physically it is questionable, as it ignores the non-vanishing permeability of vacuum, but still it can be considered as an approximation for very wide crack openings. Similar conditions have been suggested and developed by Deeg [3], Pak [4] and Suo et al. [5].The fracture behavior of a cracked piezoelectric media was studied by Sosa and Pak [6]. They found that both stresses and electric displacements have the conventional square-root singularities. Zhang and Hack [7] solved the problem of multiple cracks in an infinite piezoelectric solid. The problem of impermeable parallel cracks in piezoelectric materials by the 'pseudo-traction-electric displacement' method was studied by Han and Chen [8]. The solution of multiple mode-I cracks problem in piezoelectric materials was obtained by Han and Wang [9] using distributed dislocation technique. A failure criterion

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http://dx.doi.org/10.1016/j.engfracmech.2017.05.031
0013-7944/

for multiple permeable cracks embedded in an infinite piezoelectric solid using the equivalent inclusion method was proposed by Chao and Huang [10]. Gao [11] solved a generalized two-dimensional problem in piezoelectric media with defects, by using exact boundary conditions. Xu and Rajapakse [12] found that the exact electric boundary conditions accounting for the medium inside the crack gaps will be reduced to the impermeable crack model when the poling direction is perpendicular to the applied electric field. The exact solution of multiple collinear cracks in piezoelectric materials was obtained by Hao [13] and the permittivity of the air of crack interior was assumed. Yang [14] obtained a closed-form solution to the Mode-I fracture problem in an infinite piezoelectric material under electromechanical loading. The problem of a piezoelectric layer with a crack normal to the boundaries under in plane electro mechanical loading is investigated by Wang and Mai [15]. They studied the effects of crack size and crack position on the stress and electric displacement intensity factors for different crack face boundary conditions. The closed form solutions of electroelastic field for two anti-plane collinear cracks in a piezoelectric strip have been obtained by Li [16]. Wang and Mai [17] argued that electrical impermeabilities are reasonable for the anti-plane crack. The problem of periodic array of mode-I crack in a piezoelectric medium under electromechanical loads was analyzed by Han and Wang [18]. Zhou et al. [19] solved the problems of multiple parallel mode-I cracks in a piezoelectric solid. The interactions of multiple parallel symmetric finite length cracks in a piezoelectric material were analyzed by Zhou et al. [20]. The stress field of a single dislocation, the force between two parallel dislocations, and the stress field of various types of infinite dislocation walls and arrays were analyzed for a hexagonal crystal by Chou [21].The concept of distributed dislocation technique has been applied to the calculation of the field intensity factors of cracked piezoelectric materials. The problem of two collinear unequal cracks in a piezoelectric plane under mode I electromechanical loadings were solved by Li and Lee [22]. Asadi [23] investigated the axisymmetric crack problems in transversely isotropic piezoelectric media by means of distributed dislocation technique. The distributed dislocation technique was also applied to the static analysis of cracked functionally graded piezoelectric layer by Mousavi and Paavola [24]. The electromechanical fields at a functionally graded piezoelectric strip containing multiple moving cracks under anti-plane mechanical and in-plane electrical loadings were analyzed by Bagheri et al. [25]. The problem of multiple interfacial cracks between two bonded dissimilar piezoelectric materials under electrically impermeable or permeable conditions was analyzed by Nourazar and Ayatollahi [26].Compared to the intensive theoretical works, only a few open literatures studied an arbitrary shaped in-plane crack embedded in an infinite transversely isotropic piezoelectric material. Because piezoceramic materials are very brittle and cracked piezoelectric materials obviously contain multiple cracks with an extremely high crack density, the interaction between multiple cracks may significantly affect their fracture behavior.

The aim of the paper is to give a closed-form solution to the mixed-mode fracture problem of piezoelectric planes weakened by multiple cracks. An electric dislocation is defined and solved together with climb and glide edge dislocations in an infinite transversely isotropic piezoelectric plane. The standard Fourier transforms is adopted to solve the electroelastic dislocations for the piezoelectric plane with two types of boundary conditions. The stress and electric displacement fields are derived in closed- forms, exhibiting Cauchy singularity at the dislocations locations. Then, the distributed dislocation technique is utilized to perform a set of integral equations for the piezoelectric medium weakened by multiple cracks subjected to electromechanical loading. The integral equations are solved numerically and field intensity factors are determined. Several examples are solved for piezoelectric medium with cracks having different geometries and the interaction between cracks is examined.

2. Formulation of the problem

The problem under consideration is an infinite piezoelectric elastic medium with electroelastic dislocations. The medium is free of any mechanical and electrical loads. The dislocation is located at origin and directed to the positive x-axis. In general, the behavior of the piezoelectric material is anisotropic. Poled ceramics obey transversal isotropy with respect to the poling direction. The majority of piezoelectric sensors and actuators in use are operated at this poled state. The linear constitutive equations in terms of the elastic displacements and electric potential for in-plane problems are as follows:

$$
\begin{aligned}
\sigma_{xx}(x,y) &= c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial v}{\partial y} + e_{31}\frac{\partial \varphi}{\partial y},\\
\sigma_{yy}(x,y) &= c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial v}{\partial y} + e_{33}\frac{\partial \varphi}{\partial y},\\
\sigma_{xy}(x,y) &= c_{44}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + e_{15}\frac{\partial \varphi}{\partial x},\\
D_x(x,y) &= e_{15}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \varepsilon_{11}\frac{\partial \varphi}{\partial x},\\
D_y(x,y) &= e_{31}\frac{\partial u}{\partial x} + e_{33}\frac{\partial v}{\partial y} - \varepsilon_{33}\frac{\partial \varphi}{\partial y}.
\end{aligned}
\tag{1}
$$