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Analysis of a three-dimensional arbitrarily shaped interface crack in a one-dimensional hexagonal thermo-electro-elastic quasicrystal bi-material. Part 2: Numerical method

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ABSTRACT

The extended displacement discontinuity boundary integral equation and boundary element method are extended to analyze a three-dimensional (3D) arbitrarily shaped interface crack in a one-dimensional hexagonal, quasicrystal bi-material with both electric and thermal effects under combined phonon-phason-electric-thermal loadings. Based on the analogy with the analysis method for three-dimensional, transversely isotropic magnetoelectrothermoelastic bi-materials (Dang et al., in press) the numerical method is proposed for one-dimensional quasicrystal bi-materials. Firstly, the fundamental solutions for uniformly distributed, extended displacement discontinuities applied over a constant triangular element are obtained by integrating the fundamental solutions for unit-point extended displacement discontinuities given in Part 1 over the triangular area (Zhao et al., 2017). Secondly, in order to eliminate the oscillatory singularity near the crack front, the Delta function in the integral–differential equations is approximated by the Gaussian distribution function, and the Heaviside step function is replaced by the Error function accordingly. Thirdly, the extended stress intensity factors without oscillatory singularities and the energy release rate are all obtained in terms of the extended displacement discontinuities. At last, the extended displacement discontinuity boundary element method is proposed to validate the analytical solution. In the numerical simulation, the multi-physical behavior of an elliptical, interface crack is numerically simulated. The correctness of the proposed numerical method, the influence of the applied, combined phonon-phason-electric-thermal loadings, the material-mismatch, and the ellipticity ratio are all studied.

1. Introduction

In Part 1 [2], the analytical solution for an arbitrarily shaped interface crack in a one-dimensional (1D), hexagonal, thermo-electro-elastic, quasi-crystal (QC) bi-material subjected to combined phonon-phason-electric-thermal loadings is obtained. The fundamental solutions for the unit-point, extended displacement discontinuities (EDDs) are derived, and the corresponding boundary integral-differential equations are constructed. By analyzing the singular stress fields near

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the crack front, the extended SIFs are obtained in terms of the EDDs across the interface crack faces. Although the analytical method is theoretically useful, its application is limited in practice, as the analytical solution of explicit form only exists for an interface crack of certain shape under uniformly distributed loadings. For an arbitrarily shaped interface crack under complex combined loadings, numerical methods need to be developed to extend the application of the analytical solution.

Among various numerical methods, the displacement discontinuity method (DDM) first proposed by Crouch [3] has been extended to analyze crack problems in thermoelastic media [4], piezoelectric media [5–7], magneto-electro-elastic material [8–10] and 1D hexagonal piezoelectric QC material [11], where the conventional elastic displacement discontinuity was extended to temperature discontinuity, electric and magnetic potential discontinuities, and phason displacement discontinuity, respectively.

It is observed from Part 1 that there exists an oscillatory singularity in the interface crack problems, similar to isotropic thermoelastic bi-materials [4], which will induce the overlapping of the crack faces. However, this phenomenon is unreasonable and unrealistic, thus many researchers have attempted to remove the oscillatory singularity. For instance, Comninou [12] proposed a contact-zone model to eliminate the oscillatory singularity, which was later extended to piezoelectric media [13], magnetoelectroelastic media [14], and isotropic thermoelastic media [15], etc. Zhang and Wang [16] and Zhao et al. [10] approximated the Delta function with the Gaussian distribution function to remove the oscillatory singularity.

Based on the obtained fundamental solutions for unit-point EDDs given in Part 1 [2], the fundamental solutions for a constant triangular element are derived in this paper. The Delta function in the integral–differential equations is approximated by the Gaussian distribution function in the same manner as Zhang and Wang [16] and Zhao et al. [10] to remove the oscillatory singularity. Utilizing the obtained fundamental solutions, the boundary EDD method is proposed to study an arbitrarily shaped, interface crack in a 1D QC bi-material.

The paper is organized as follows: the statement of the crack problem is presented in Section 2, the EDD boundary integral-differential equations and the analogy relation between the 1D QC material and magnetoelectrothermoelastic materials are given in Section 3. The fundamental solutions for uniformly distributed EDDs over a constant triangular element are obtained based on the analogy relation, and the oscillatory singularity is removed by approximating the Delta function with the Gaussian distribution function in Section 4. In Section 5, the extended stress intensity factors (SIFs) and energy release rate (ERR) are derived in terms of the EDDs. In Section 6, the EDD boundary element method is proposed to simulate an elliptical interface crack as an example. In Section 7, the correctness of the analytical solution and numerical method is validated. In addition, the influence of the applied combined loadings, the material-mismatch and ellipticity ratio on the fracture behavior are investigated. The concluding remarks are drawn in Section 8.

2. Statement of the problem

Consider a 1D, hexagonal, QC bi-material with the interface being parallel to the periodic plane. A Cartesian coordinate system is set up with the *xoy* plane coinciding with the interface, and the quasi-periodic direction is along the *z*-direction. The two perfectly bonded, dissimilar solids are assumed to occupy the upper and lower half-spaces, which are denoted as material 1 and 2, respectively. An arbitrarily shaped, interface crack *S* lies in the interface plane *xoy*, and the upper and lower surfaces of crack *S* are denoted by S^+ and S^-, respectively. The uniformly distributed, combined loadings including phonon mechanical loading p_0, phason mechanical loading h_0, steady heat flux q_0, and electric displacement component ω are applied at infinity, as schematically shown in Fig. 1. If the adhesion between the solids outside the crack is assumed to be perfect, the extended displacement and stress components will be continuous along the interface outside the crack *S*. The outer normal vectors of S^+ and S^- are expressed as

$$\{n_i\}^+ = \{0, 0, -1\}, \quad \{n_i\}^- = \{0, 0, 1\}. \tag{1}$$

As is well known, the electric and thermal boundary conditions on the crack face can be diverse. In this paper, the electrically impermeable and thermally insulated boundary condition along the crack face is adopted. By superposition, the present interface crack under far field loadings is equivalent to a perturbed problem in fracture mechanics, where the interface crack is applied extended loadings on the upper and lower crack faces with the same magnitude but opposite directions. The perturbed problem is now the focus of this paper.

3. EDD boundary integral-differential equation method

As is discussed in Part 1 [2], the extended stresses at an arbitrary field point on the interface crack *S* can be expressed in terms of the EDDs across the interface crack faces. The corresponding boundary integral-differential equations are

$$\begin{aligned}\sigma_{zx}(x,y) = \int_{S^+} \Big\{ & [L_{11}(3\cos^2\varphi - 1) - L_{12}(3\sin^2\varphi - 1)]\tfrac{1}{r^3}\|u_x(\xi,\eta)\| \\ & + 3(L_{11}+L_{12})\tfrac{\sin\varphi\cos\varphi}{r^3}\|u_y(\xi,\eta)\| - L_{13}\tfrac{\cos\varphi}{r^2}\|\theta(\xi,\eta)\|\Big\} dS(\xi,\eta) \\ & - 2\pi\left(L_{14}\tfrac{\partial\|u_z(x,y)\|}{\partial x} + L_{15}\tfrac{\partial\|\phi(x,y)\|}{\partial x} + L_{16}\tfrac{\partial\|w_z(x,y)\|}{\partial x}\right),\end{aligned} \tag{2a}$$