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Theoretical and numerical prediction of crack path in the material with anisotropic fracture toughness

Yue Gao, Zhanli Liu *, Qinglei Zeng, Tao Wang, Zhuo Zhuang, Keh-Chih Hwang

AML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
Center for Mechanics and Materials, Tsinghua University, Beijing 100084, China

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ABSTRACT

The crack path in the anisotropic medium is studied theoretically and numerically in this paper, with focusing on the effects of the anisotropic fracture toughness. A weak plane model is adopted to characterize the anisotropic fracture toughness and the maximum energy release rate criterion (MERR) is chosen to predict the crack path. We prove that the crack deflecting direction in a weak plane material, theoretically, only relates to the weak plane direction, the weakness of the plane, and the ratio of stress intensity factors before crack extending. Two critical weakness ratios are found: one is for that the weak plane captures all cracks for any loading state; the other is for that the weak plane traps the crack once it is deflected to the plane. To further numerically study the complex crack path in an anisotropic medium, the extended finite element method (XFEM) embedded MERR criterion and weak plane model is developed, in which a mesh independent piecewise linear crack formulation is proposed to capture the curved crack path. By modeling the three point bending loading test, the influences of the weak plane angles and the weaknesses on crack path deflection are studied, and periodically oscillatory crack path behaviors are found as observed in experiments. Further more, the crack extending through multiple layers of weak plane material and isotropic material is numerically studied, which shows that the crack diffracts at the material interface, and the crack path in the anisotropic toughness material is rougher than in the isotropic material. The presented work in this paper will be helpful to understand and control the crack path in the rocks as well as other anisotropic materials.

1. Introduction

Anisotropic materials are commonly used in the world, such as rocks, crystals, and many other nature materials. However, their fracture behaviors have not been fully understood. In this article, we will focus on the crack path in the anisotropic material, and demonstrate a theoretical prediction of it with numerical verifications, then study the complex fracture behavior in anisotropic material numerically.

When we state that a medium is anisotropic, we usually mean that the constitutive model in the medium is anisotropic. However, there is another type of anisotropy, the anisotropic fracture toughness. Nasseri and Mohanty [1] measured fracture

* Corresponding author at: Center for Mechanics and Materials, Tsinghua University, Beijing 100084, China; AML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China.

E-mail address: liuzhanli@tsinghua.edu.cn (Z. Liu).

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toughness for granitic rocks in different directions. They found that orientations of the grain axis and microcracks might be various in different faces of a rock specimen. Kataoka et al. [2] measured two types of anisotropic rocks, African granodiorite and Korean granite. They found that, the fracture toughness might be 29% larger in the direction perpendicular to the bedding plane than parallel to it. Chandler et al. [3] measured Mancos shale, and they found that the fracture toughness is 0.72 MPa $m^{1/2}$ when the crack is normal to the bed plane of shale, but only 0.21 MPa $m^{1/2}$ when parallel to the bed plane in their specimen. Besides rocks, the anisotropic fracture toughness are also widely observed in other materials, e.g., NiAl single crystals [4], graphene aerogels infiltrated with epoxy resin [5], microcellular foamed ABS polymer [6], polyurethane and polyisocyanurate foams [7].

Compared with isotropic material, the fracture paths in the material or composite structures with anisotropic fracture toughness are much more complicated, but also more interesting. Takei et al. [8] performed tearing experiments on polypropylene sheets, and validated the anisotropic fracture toughness model. They pointed out that for a well-designed fracture toughness profile, there would be forbidden directions for crack extending. Audoly et al. [9] and Reis et al. [10] observed an oscillatory fracture when a cylinder cuts a brittle elastic film in experiments. Nam et al. [11] showed results of controlled crack paths in a multilayer medium, which is combined with a silicon nitride Si_3N_4 film, a single-crystalline silicon substrate, and a silicon wafer. Three different crack path patterns were found: straight, oscillatory, and orderly bifurcated, by controlling the system parameters. Nasseri and Mohanty [1] and Kataoka et al. [2] also found that the cracks would extend more roughly in the larger fracture toughness direction, when they measured the fracture toughness of granitic rocks in laboratory.

There have already been some theoretical and numerical works to predict the crack path in anisotropic materials. Theoretically, plenty of criteria are developed to predict crack extending direction, such as (1) the principle of local symmetry ($K_{II} = 0$), (2) maximum K_I criterion, (3) maximum hoop stress criterion, (4) maximum energy release rate (MERR) criterion. These criteria have been studied well in the isotropic constitutive model, and they usually give the same prediction in an isotropic medium. However, in an anisotropic medium, the crack extending directions predicted by them are usually different [12]. There are many people who endorsed the MERR criterion, and warned that other criteria may give incorrect predictions even in isotropic materials [e.g., 8,12–15]. Zeng and Wei [16] used MERR to analyze the initiation of hydraulic fracture deflection when it crosses a given nature fracture with weak strength, based on the study of He and Hutchinson [17] and Hutchinson and Suo [18]. The MERR criterion is generally accepted in anisotropic material fracture study [3,19], and is adopted in this paper. Recently, Li et al. [20] also developed a microplane constitutive model to consider the anisotropic mechanics behavior of shale rocks.

From the aspect of numerical simulation, phase field fracture model is often used on the anisotropic fracture toughness topic. Hakim and Karma [12,14] demonstrated a force-balance principle (FL) and compared the results with maximum energy release rate criterion (MERR). They found out that the kink angle in their phase field simulation result was between the predictions of FL and MERR. Li et al. [19] focused on the anisotropic fracture toughness that has rotational symmetry of order four, which looks like a flower type profile. A zigzag crack extending between two constrained edges is simulated. Shanthraj et al. [21] developed a finite-strain anisotropic phase field model to study the kinking and branching of the crack path resulting from the crystallographic misorientation across the laminate boundary and the grain boundaries. Though phase field method is widely used in modeling complex fracture phenomenon recently, it is hard to extract the fracture parameters from the phase field modeling. Li et al. [22] formulated a discrete micromechanical approach based on the lattice discrete particle model to simulate the anisotropic mechanical behavior of shale.

The extended finite element method (XFEM) is a powerful tool in numerical fracture modeling [23–27]. With this method, mesh refinement is not required when crack extends, and stress intensity factors are easy to be obtained with interaction integrals [28]. To the best knowledge of the authors, there is yet no work dealing with the material with anisotropic fracture toughness using XFEM.

The *weak plane model* is introduced in this paper as a special case of the anisotropic fracture toughness models. The model assumes that the fracture toughness is the same in almost every direction as an isotropic material, but is lower in a specific direction, which is the direction of the weak plane. The sedimentary rock, like shale, is the typical material which obeys the weak plane model and will be of particular interest in this paper.

In this study, the MERR criterion is adopted to predict the crack path in an anisotropic medium, with the weak plane model to simulate the anisotropic fracture toughness. A simple crack deflection criterion is demonstrated from the weak plane model, and the forbidden areas phenomenon of crack deflection is discussed. An XFEM based numerical modeling technique, which embeds the MERR criterion and the weak plane model to handle the anisotropic fracture toughness, is developed to study the complex crack path in anisotropic material, and the results between theoretical prediction and numerical simulation are compared.

2. Theoretical model

2.1. Maximum energy release rate criterion

The MERR criterion is adopted in this paper to predict crack extending direction, and is briefly introduced in this section.