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Topological derivative-based fracture modelling in brittle materials: A phenomenological approach

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ABSTRACT

The Griffith-Francfort-Marigo damage model describes the behavior of brittle materials under the quasi-static loading assumption, focusing on the evolution of damage regions. It is based on the minimization of a shape functional given by the sum of the total potential energy of the system with a Griffith-type dissipated energy, with respect to the distribution of the healthy and damaged phases, under an irreversibility constraint. A natural approach to deal with such a minimization problem consists in considering the topological derivative concept to nucleate small damaged regions and the shape gradient to propagate them. In contrast to such an approach, in this paper the Griffith-Francfort-Marigo damage model is revisited by using the sole tool of topological derivative. In particular, we propose a striking simple numerical scheme based on the computation of the topological derivative field to determine damage nucleation as well as crack/damage propagation. In other words, the topological derivative is used as descent direction to minimize the Francfort-Marigo functional indicating, in each iteration, the regions that have to be damaged. Therefore, the proposed topology optimization algorithm is able to capture the whole nucleation and propagation damaging process, including important features like kinking and bifurcations. These properties are confirmed through several numerical experiments and by comparison with available laboratory experiments.

1. Introduction

Many works in Fracture Mechanics address the issue of microscopic modelling of fractures and the coupling of some defect atomistic models with macroscopic elasto-plastic models. In this paper, we focus on a purely macroscopic model in the framework of continuum mechanics. Roughly speaking, continuum models can be classified in two main categories. On the one hand, there are models of crack growth and propagation which assume that the crack is a surface evolving in three-dimensional body, with specific evolution laws, which are found innumerable in Fracture literature (often depending on the body shape and dimensions). On the other hand, one can consider models of fracture, where the crack is identified as a thin damage. In this case there exists a competition between the initial healthy elastic phase and another damaged elastic

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phase. The transition from healthy to damaged can be smooth or sharp, i.e., there is an interface between a healthy and a fully damaged zone. Our model belongs to this second class.

The origin of such a model amounts to the British engineer A.A. Griffith in 1921, who published a paper on fracture of glass. In this work, Griffith assumes that flaws pre-exist in the body, where stress concentrates, provoking atomic debonding and resulting in crack propagation, until the body breaks. The pre-existing crack is submitted to an external load: force or imposed displacement. In Griffith's model, the system is modeled by two thermodynamic variables: the area of the crack and the displacement of the loading grips. The energy of the system is the sum of the elastic energy in the body, and the surface energy of the crack, and is a function of a single thermodynamic variable: the area of the crack. When the crack evolves, the stress in the sample is partially relieved, so that the elastic energy is reduced. At the same time, the advancing crack creates more surface area, so that the surface energy increases. Thermodynamics dictates that the process should go in the direction that reduces the total available free energy. If the decrease in elastic energy prevails, the crack grows, otherwise the crack heals. Specifically, let U be the internal energy (i.e., the macroscopic energy of the atoms), P be the work of the volume and surface forces (i.e., $-P$ is the potential energy), S be the body entropy and T_0 the surface temperature, assumed constant and equal to the ambient temperature. Let us assume that the process is quasi-static, the kinetic energy and the volume heat sources are negligible, whereas the surface heat supply must not vanish a priori (recall that boundary loads are prescribed). Therefore the combination of the first and second laws of Thermodynamics yields

$$\frac{d}{dt}(U - P - T_0 S) \leqslant 0, \tag{1.1}$$

where Griffith takes U as the sum of the stored elastic energy E and a surface term proportional to the crack area, D. Thus, Griffith's Law (1.1) strictly tells us that the available free energy must decrease in time, that is that the total energy $\mathcal{F} := E - P + D$ tends to be minimized, while the entropy S increases.

A stronger postulate was considered about 70 years later by Francfort and Marigo [12] when revisiting Griffith's model under the framework of global minimization of the energy. Indeed, the authors, and after them a series of coworkers and contributors did suppose that at each quasi-static step, the total energy $\mathcal{F}$ achieves the global minimum with respect to the distribution of the healthy and damaged phases. Furthermore, they assumed that the crack is irreversible, meaning that healing is precluded: at each step, either the crack is unchanged, and hence load is increased, either the crack advances, and hence its area is strictly increasing.

There are several ways to compute the minimum of the energy in order to provide a computational algorithm of fracture/-damage propagation (see, e.g., [11]). One minimization scheme suggested in [1] relies on shape optimization principles. It consists of a descent method driven by the shape gradient of the energy functional, i.e., the energy decreases in the normal direction to the boundary of the damage region with a magnitude given by the shape derivative of $\mathcal{F}$. Furthermore, in order to nucleate new damage regions, the so-called topological derivative of $\mathcal{F}$ was also considered in [1]. Let us emphasize that in theory the concepts of shape and topological derivatives are distinct, and the latter is computed in the undamaged part of the body, in order to determine if it is energetically worth to create some new damage away from the existing one. One drawback of using the shape gradient approach, is in fact that it is a vector field concentrated on the boundary of the damage, as opposed to the topological derivative which is a scalar field distributed in the whole domain. Therefore, one needs a very good computation of the normal vector to the damage region, because this vector will determine the crack/damage path. It turns out that the shape optimization method of [1] was promising, but computationally expensive.

Topological sensitivity analysis may be considered for a pure fracture model as in [19,16] as for a damage model with crack-like damage regions. It is the purpose of the present work to revisit Griffith-Francfort-Marigo damage model by using solely the topological derivative concept, that is, the computation of this scalar quantity should allow us to determine damage nucleation as well as crack/damage propagation, relying on the contour lines of the topological derivative field. It can be proven, but is not the aim of this work, that from a theoretical standpoint the concepts of shape and topological derivatives do coincide on the boundary of the damage region [6]. In this work, we present a simple numerical scheme that was able to improve the results of [1], not only in terms of computational cost but also in terms of successful crack propagation assessment tests. The interest of this method is its striking simplicity: to achieve minimization, a single scalar field is computed from which nucleation and propagation of damages are determined. In particular, the topological derivative is used as descent direction to minimize the Francfort-Marigo functional indicating, in each iteration, the regions that have to be damaged. Therefore, the proposed topology optimization algorithm is able to capture the whole nucleation and propagation damaging process, including important features like kinking and bifurcations. These properties are confirmed through several numerical experiments, whose results are compared with real laboratory tests when available. Let us emphasize however that being a descent method, what is actually achieved is local rather than global minimization, which is also more sound from a Physical perspective. In this respect, our choice has been to refine the mesh at the crack tip as soon as a local advance is made. In such a way, according to our numerical results, bifurcation and kinking are rather well captured.

The paper is organized as follows. The Griffith-Francfort-Marigo damage model is revisited in Section 2. Its associated topological derivative is presented in Section 3. The resulting topology optimization algorithm is shown in all its details through Section 4. The obtained numerical results are presented in Section 5, where the whole nucleation and propagation damaging process is observed, together with important features such as kinking and bifurcations. Finally, the paper ends with some concluding remarks in Section 6.