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Dynamic non-local theory solution to a permeable mode-I crack in a piezoelectric medium

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A B S T R A C T

In the present paper, the non-local theory, the generalized Almansi's theorem and the Schmidt method are developed for the analysis of a permeable mode-I crack in a piezoelectric medium(PZT-4, P-7, PZT-5H) under the harmonic stress waves. The problem is formulated through Fourier transformation into two pairs of dual integral equations, in which the unknown variables are the displacement jumps across the crack surfaces. For solving the dual integral equations, the displacement jumps across the crack surfaces are directly expanded as a series of Jacobi polynomials. The dynamic non-local stress and the dynamic non-local electric displacement are obtained at the crack tips. Numerical examples are provided to show the effects of the crack length, the characteristics of the harmonic wave and the lattice parameter on the dynamic stress field and the dynamic electric displacement field near the crack tips in a piezoelectric medium. Different from the classical solutions, the present solution exhibits no stress and electric displacement singularities at the crack tips in a piezoelectric medium.

1. Introduction

The excellent electromechanical coupling effect of the piezoelectric medium has been widely used in various engineering fields, such as smart structures and systems. The fracture problem of piezoelectric medium is important for practical applications. Transient response analysis of a Mode III interface crack between a piezoelectric layer and a functionally graded orthotropic material layer is conducted by Shin and Kim [1] using integral transform techniques. Bhargava and Verma [2] proposed a strip-electro-mechanical model for two semi-permeable collinear cracks, symmetrically situated and transversely oriented in a poled piezoelectric strip. Hu et al. [3] proposed a moving Dugdale interfacial crack model, and the interfacial crack between dissimilar piezoelectric materials under anti-plane electro-mechanical loading. Chen et al. [4] investigated the mode-I transient response of a semi-infinite conducting crack propagating in a piezoelectric material with hexagonal symmetry under normal impact loading. More about the fracture behaviors of piezoelectric medium have been reported by [5–8].

All above works were studied for fracture problem based on classical elastic theory and there exists stress singularity at the crack tips. According to the physical nature in engineering, the stress field near the crack tips should be finite. Beginning with Eringen [9], the non-local theory was used to study the fracture problem in elastic materials. Overcoming the stress singularity at the crack tips in the classical elastic theory, Eringen et al. [10] and Eringen [11,12] analyzed the stress near

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the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension, shear and anti-plane shear loading by using non-local theory. In recently, Zhou et al. [13,14] analyzed some static fracture problems in piezoelectric materials and magnetoelectroelastic materials by using non-local theory and the Schmidt method [15]. Jamia et al. [16] investigated the static problem of two collinear mixed-mode limited-permeable cracks in a functionally graded piezoelectric medium by using non-local theory. Liu et al. [17] studied the thermo-electro-mechanical free vibration of piezoelectric nanoplates based the nonlocal theory and Kirchhoff theory. To the best of author knowledge, the dynamic electro-elastic behavior of a permeable crack in piezoelectric medium under the harmonic stress waves has not been studied by means of non-local theory.

In the present paper, the dynamic non-local theory solution of a permeable crack in piezoelectric medium subjected to harmonic stress waves is studied by using the generalized Almansi's theorem and the Schmidt method. In Section 2, the governing equations and boundary conditions are given and the considered dynamic fracture problem is described. In Section 3, the two pairs of dual integral equations are derived. The dynamic non-local stress and dynamic non-local electric displacement fields are obtained in Section 4. In Section 5, the numerical examples based on the analytical solutions are discussed in details. The conclusions are stated in Section 6.

2. Formulation of the problem

As shown in Fig. 1, a mode-I crack of length $2l$ along x -axis in a piezoelectric medium is considered. Let ω be the circular frequency of the incident wave. $u_0^{(j)}(x,z,t)$ and $w_0^{(j)}(x,z,t)$ are the mechanical displacement in the $x-$ and $z-$ directions; $\phi_0^{(j)}(x,z,t)$ is the electric potential. $\sigma_{kz0}^{*(j)}(x,z,t)$ and $D_{k0}^{*(j)}(x,z,t)(k=x,z,\ j=1,2)$ are the non-local stress field and in-plane non-local electric displacement field, respectively. It should be noted that all the quantities with superscript $j(j=1,2)$ correspond to the upper half plane 1 and the lower half plane 2 as shown in Fig. 1. Duo to the incident waves are harmonic stress waves, all field quantities of $u_0^{(j)}(x,z,t)$, $w_0^{(j)}(x,z,t)$, $\phi_0^{(j)}(x,z,t)$, $\sigma_{kz0}^{*(j)}(x,z,t)$ and $D_{k0}^{*(j)}(x,z,t)$ can be assumed as

$$[u_0^{(j)}(x,z,t), v_0^{(j)}(x,z,t), \phi_0^{(j)}(x,z,t), \sigma_{kz0}^{*(j)}(x,z,t), D_{k0}^{*(j)}(x,z,t)] = [u^{(j)}(x,z), w^{(j)}(x,z), \phi^{(j)}(x,z), \sigma_{kz}^{*(j)}(x,z), D_k^{*(j)}(x,z)]e^{-i\omega t} \quad (1)$$

In what follows, the time dependence of $e^{-i\omega t}$ will be suppressed but understood.

2.1. Boundary conditions

As discussed in Parton [18], the electric permeable crack model is adopted in the present paper. Therefore, the mixed boundary conditions for the problem can be given as

$$\begin{cases} \sigma_{xz}^{*(1)}(x,0^+) = \sigma_{xz}^{*(2)}(x,0^-) = 0, & \sigma_{zz}^{*(1)}(x,0^+) = \sigma_{zz}^{*(2)}(x,0^-) = -\sigma_0 \\ \phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-), & D_z^{*(1)}(x,0^+) = D_z^{*(2)}(x,0^-) \end{cases}, \quad |x| \leqslant l \quad (2)$$

$$\begin{cases} u^{(1)}(x,0^+) = u^{(2)}(x,0^-), & w^{(1)}(x,0^+) = w^{(2)}(x,0^-) \\ \sigma_{zz}^{*(1)}(x,0^+) = \sigma_{zz}^{*(2)}(x,0^-), \sigma_{xz}^{*(1)}(x,0^+) = \sigma_{xz}^{*(2)}(x,0^-) & \\ \phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-), & D_z^{*(1)}(x,0^+) = D_z^{*(2)}(x,0^-) \end{cases}, \quad |x| > l \quad (3)$$

$$u^{(j)}(x,z) = w^{(j)}(x,z) = \phi^{(j)}(x,z) = 0, \sqrt{x^2+z^2} \to \infty \quad (4)$$

where σ_0 is the magnitude of the incident waves.

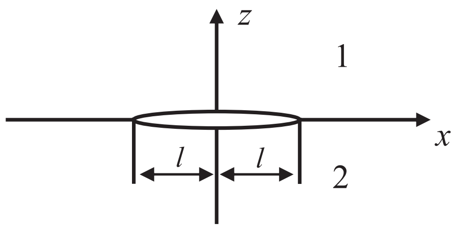

Fig. 1. The coordinate system for a crack in a piezoelectric medium plane.