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Accuracy and robustness of stress intensity factor extraction methods for the generalized/eXtended Finite Element Method

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ABSTRACT

This paper investigates the accuracy and robustness of three Stress Intensity Factor (SIF) extraction methods for the Generalized/eXtended Finite Element Method (G/XFEM): the Cutoff Function Method (CFM), the Contour Integral Method (CIM), and the Displacement Correlation Method (DCM). Challenges in SIF extraction from G/XFEM solutions using energy-based methods such as the CFM and the Interaction Integral Method are discussed. Numerical studies on problems with stress-free crack surfaces show that the CFM is path independent while the CIM is not. They also show that unless a proper enrichment scheme is adopted, SIFs extracted with the DCM are of low accuracy and sensitive to extraction parameters. Strategies to address both issues with the DCM are presented. This paper demonstrates that the DCM, with proper enrichment of the G/XFEM approximation, has an accuracy and robustness comparable to the CFM at a fraction of the computational cost. It is also shown that the DCM can be applied to problems where SIF extraction using domain integrals is not possible. Several problems aimed at investigating the applicability and accuracy of the various extraction methods are solved.

1. Introduction

Stress Intensity Factors (SIFs) are fundamentally important parameters for linear elastic fracture mechanics problems. They are used for example, to predict the crack growth speed and direction, and whether a crack will propagate or not under a given load. When used with an exponential equation, like Paris law [28], to predict crack propagation speed, a small error in SIFs may lead to a large discrepancy in the predicted fatigue life of a structural component. Therefore, SIFs must be computed with high accuracy.

There are several approaches for the extraction of SIFs from numerical approximations of the solution of linear elasticity equations. Extraction methods based on energy release rate concepts include the J-Integral [50], the Interaction Integral Method (IIM)[36], the Contour Integral Method (CIM) [54,57], and the Cutoff Function Method (CFM) [57]. A review of methods for calculating energy release rates can be found in [30]. These methods are accurate since theoretically they converge at the same rate as the strain energy [4], which is the quantity with the highest convergence rate in a finite element analysis. Assumptions about the crack geometry in three-dimensional problems are usually adopted in their numerical implementation [19]. This in general leads to a loss of accuracy in the case of non-planar 3-D fractures [19].

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Nomenclature

$\mathcal{I}(\alpha)$	index of all enrichment functions at node α
Ω	problem domain
(r, θ, x_3)	cylindrical coordinate system at crack front
(x, y, z)	global Cartesian coordinate system
(x_1, x_2, x_3)	crack front Cartesian coordinate system
α	index of a node in a finite element mesh
$\bar{\boldsymbol{T}}^{(\bar{\boldsymbol{v}}^{-III})}$	traction vector computed from $\bar{\boldsymbol{v}}^{-III}$
$\bar{\boldsymbol{T}}^{(\bar{\boldsymbol{v}}^{-II})}$	traction vector computed from $\bar{\boldsymbol{v}}^{-II}$
$\bar{\boldsymbol{T}}^{(\bar{\boldsymbol{v}}^{-I})}$	traction vector computed from $\bar{\boldsymbol{v}}^{-I}$
$\bar{\boldsymbol{T}}^{(\bar{\boldsymbol{u}})}$	traction vector computed from $\bar{\boldsymbol{u}}$
$\bar{\omega}_\alpha$	the support of shape function N_α
$\bar{\phi}(r, x_3)$	cutoff function dependent on r and x_3 in local cylindrical system (r, θ, x_3)
$\bar{\phi}_r(r)$	cutoff function dependent on r in local cylindrical system (r, θ, x_3)
$\bar{\phi}_{x_3}(x_3)$	cutoff function dependent on x_3 in local cylindrical system (r, θ, x_3)
$\bar{\rho}$	extraction parameter for an extraction method
$\bar{\boldsymbol{u}}(r, \theta, x_3)$	displacement vector in the crack front Cartesian coordinate system, written in terms of cylindrical coordinates (r, θ, x_3)
$\bar{\boldsymbol{v}}^{-III}$	CIM extraction function for K_{III}
$\bar{\boldsymbol{v}}^{-II}$	CIM extraction function for K_{II}
$\bar{\boldsymbol{v}}^{-I}$	CIM extraction function for K_I
$\bar{\boldsymbol{w}}^{-III}$	CFM extraction function for K_{III}
$\bar{\boldsymbol{w}}^{-II}$	CFM extraction function for K_{II}
$\bar{\boldsymbol{w}}^{-I}$	CFM extraction function for K_I
$\boldsymbol{p}^3$	prescribed traction on Γ_3^L
$\boldsymbol{p}^4$	prescribed traction on Γ_4^L
$\boldsymbol{u}(x_1, x_2, x_3)$	displacement vector in the crack front Cartesian coordinate system (x_1, x_2, x_3)
$\boldsymbol{x}_\alpha$	coordinates of a node in a finite element mesh
$\mathcal{H}$	generalized Heaviside function
χ_α	basis of the local space associated with node α
Δ_k	distance between sampling points with distance r_k and r_{k+1} to the crack front
$\Gamma_1^L,\ \Gamma_2^L,\ \Gamma_3^L,\ \Gamma_4^L$	portions of the boundary of Ω_s^L
$\hat{K}_i^j$	reference SIF for Mode i at the crack front vertex j
κ	Kolosov constant
$[\![\bar{\boldsymbol{u}}]\!]$	displacement jump across crack surface in the crack front Cartesian coordinate system, written in terms of cylindrical coordinates (r, θ, x_3)
$[\![\bar{u}_1]\!]$	displacement jump in the crack front coordinate system direction x_1
$[\![\bar{u}_2]\!]$	displacement jump in the crack front coordinate system direction x_2
$[\![\bar{u}_3]\!]$	displacement jump in the crack front coordinate system direction x_3
$[\![\boldsymbol{u}]\!]$	displacement jump across the crack surface written in the crack front Cartesian coordinate system (x_1, x_2, x_3)
$[\![\boldsymbol{u}^h]\!]$	G/XFEM approximation of displacement jump $[\![\boldsymbol{u}]\!]$
$[\![L_{\alpha i}]\!]$	jump of an enrichment function
$\mathbb{S}_{ENR}$	enrichment space
$\mathbb{S}_{FEM}$	FEM approximation space
$\mathbb{S}_{GFEM}$	G/XFEM approximation space
$\mathbf{H}_L^\alpha$	set of Heaviside enrichments at node α
$\mathbf{L}^{\mathrm{OD}}_{\mathrm{front}-x_1}$	OD enrichment functions in direction x_1
$\mathbf{L}^{\mathrm{OD}}_{\mathrm{front}-x_2}$	OD enrichment functions in direction x_2
$\mathbf{L}^{\mathrm{OD}}_{\mathrm{front}-x_3}$	OD enrichment functions in direction x_3
$\mathcal{K}$	curvature of the crack front
$\underline{\tilde{\boldsymbol{u}}}_{\alpha i}$	G/XFEM degrees of freedom
$\underline{\hat{\boldsymbol{u}}}_\alpha$	standard FEM degrees of freedom
ν	Poisson's ratio
Ω_S	extraction domain for CFM
Ω_s^L	extraction domain for CIM
$\phi_{\alpha i}$	a G/XFEM shape function
ρ_1	inner radius of extraction domain for the CFM or extraction path for the CIM
ρ_2	outer radius of extraction domain for the CFM or extraction path for the CIM
$\tilde{\boldsymbol{u}}$	displacement vector in global coordinate system (x, y, z)