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Weight function determinations for shear cracks in reinforced concrete beams based on finite element method

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ABSTRACT

An efficient approach of evaluating the weight functions for the generally load applied faces of shear cracked RC beams with varying shear span/beam depth ratios is presented using the corresponding weight functions for the RC beams with two shear span/beam depth ratios. The shear cracked RC beam under applied load can be considered as an oblique edge crack elastic geometry under mixed load conditions if the nonlinear bridging force is evaluated according to a crack bridging model separately. With pure bend load conditions, both Mode I and II weight functions for the oblique edge crack geometries with three shear span/beam depth ratios for different crack length/beam depth ratios are evaluated through applying the Virtual Crack Extension technique with symmetric mesh around the crack-tip into finite element analysis, where all weigh function components exhibit orderly trends with respect to the changing shear span/beam depth. As a result, the weight functions for the shear crack RC beams with the other shear span/beam depth ratios can be calculated following the orderly trends. Since the crack-tip singular behaviors of fracture problems is observed only in the primary crack-face nodal weight functions, to facilitate application, the primary crack-face weight functions are formulated accurately as a function of crack length and distance away from the crack-tip employing the fitting and interpolation methods.

1. Introduction

The oblique shear failure of reinforced concrete beams has long been known to be a brittle and catastrophic type of failure. Even though the behavior of reinforced concrete in shear has been studied for more than 100 years, the problem of how shear failures occur in RC beams still remains, by and large, unsolved as international codes, such as the American Concrete Institute (ACI) code [1] or the Eurocode 2 [13], are based on rather (semi-) empirical considerations.

The fracture mechanics provides a possible theoretical approach of analyzing the oblique shear crack of RC beams. Unfortunately, the concrete fracture process is complicated because a nonlinear strain softening, i.e., a negative slope in the stress-deformation diagram, is observed due to the localized crack process zone, which makes the Linear Elastic Fracture Mechanics (LEFM) seems inapplicable. However, the application of LEFM is not hindered at all by the seemingly nonlinear characteristics of the concrete cracking behavior following the idea of a fictitious crack mode proposed by [11], where the nonlinear crack bridging effect is simplified as a decreasing function of the displacement discontinuity separately. In terms of the crack

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bridging model, several versatile models have been proposed based on the cohesive law [5] or through fitting experimental data [24].

In LEFM, the weight function concept which was firstly proposed by [4,22] processes very strong advantages because the stress intensity factors for any arbitrary state of loading can be determined if the weight function of a given crack geometry is evaluated from a (perhaps simple) reference state of loading owing to load-independent characteristics of weight functions. Therefore, the use of weight function can obviate the repeated computer calculations of the stress intensity factors, which makes it suitable for some purposes, such as the structural life prediction where it is indispensable to analyze the cyclic load changes of a flawed structure. In addition, the weight function facilitated the formulation of the problem of a bridged crack in an elastic medium as an integral equation [7] illustrating the relation between crack opening displacement and crack bridging tractions. The integral equation was further exploited in determining the crack bridging forces [18,19].

Unfortunately, until now, the number of fracture problems with a close form analytical solution of weight function is very small. Because of that, different numerical methods suitable for the determination of weight functions have been presented, one of which is the Finite Element Method (FEM). The Virtual Crack Extension (VCE) technique, as suggested by [10,21], provides an efficient finite element calculation of stress intensity factors and nodal weight functions. This technique has been employed in determining the 2-D Mode I weight functions in [8] and extended to 2-D Mixed Mode fracture problems in [9] through the use of symmetric mesh in the vicinity of crack tip. Obviously, the oblique shear crack of RC beams can be simply considered as 2-D Mixed Mode fracture problems.

For the failure pattern of RC beams, one of the decisive factors is the size effect as reported in [3,26]. Muttoni and Ruiz [17] and Zararis and Papadakis [25] concluded that the shear crack leading to failure occurs only in beam with a shear span to depth ration from 2.5 to 8.0. Thus, weight functions for beam geometries with varying sizes are necessary, whereas the weight functions depend on the beam geometry. Therefore, this study is dedicate to presenting an approach of weight function determination for the shear cracked RC beams with varying shear span/beam depth ratio using only weight functions for beams with few shear span to beam depth ratios.

2. Formulation for mixed fracture mode

Exploiting the VCE technique in finite element method, the nodal weight functions for Mode I 2-D crack problems was represented in the displacement differentiation form according to the physical meaning of weight function, which is the normalized rate of change of displacements due to a unit change in the crack length for a reference state of loading, in [22]. This computationally efficient finite element methodology for Mode I cracks was extended to mixed mode cracks, with combined tension and shear loading conditions, in [9] through the use of symmetric mesh in the crack tip neighborhood. The symmetric mesh provides the decoupling characteristic of the stress, strain, displacement and traction field parameters into Mode I and Mode II components with respect to x axis in the crack tip neighborhood as shown in Fig. 1, as a result, the stress intensity factors and nodal weight functions are separated into Mode I and Mode II components. The decoupled nodal weight functions for Mode I and Mode II at i's nodal location (x_i, y_i) with crack length (a) and inclination angle (β) can be represented in the displacement differentiation form as

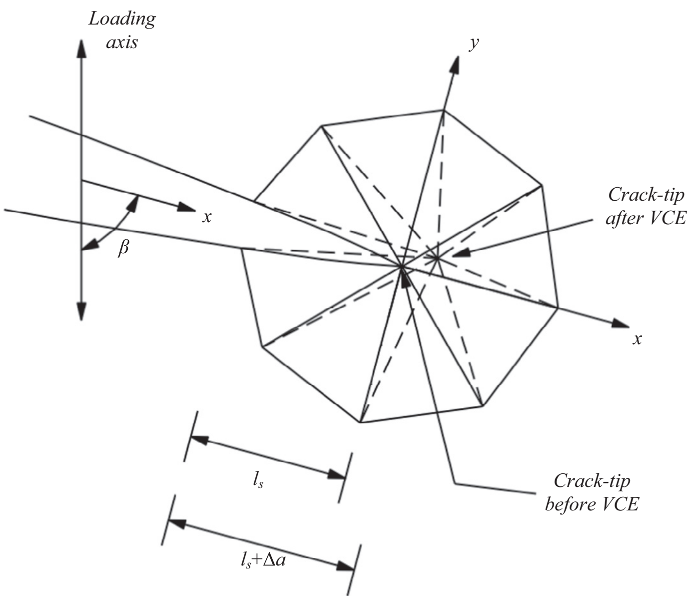

Fig. 1. Symmetric mesh in crack-tip neighborhood with respect to the global x axis.