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An XFEM-based method with reduction technique for modeling hydraulic fracture propagation in formations containing frictional natural fractures

Fang Shi [a], Xiaolong Wang [a], Chuang Liu [a], He Liu [b], Hengan Wu [a,*]

[a] CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230027, China
[b] PetroChina Research Institute of Petroleum Exploration & Development, 20 Xueyuan Road, Haidian District, Beijing 100083, China

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ABSTRACT

Field-scale modeling of hydraulic fracture development in formations containing pre-existing fractures is a time-consuming task for XFEM-based numerical models. Because the costly solving of large-scale linear equation systems has to be performed for many times during two types of iteration processes of solving the fluid-solid coupling equations and determining the contact status between frictional fracture surfaces. In view of this challenge, a reduction technique is proposed in a tightly coupled model in which the equilibrium and flow continuity equations are solved simultaneously by the Newton-Raphson method. By retaining the enriched degrees of freedom (DOFs) and removing the standard DOFs which have no contribution to fracture opening, the dimensions of linear equation systems to be solved for both the fluid-solid coupling iteration and the nonlinear contact iteration can be significantly reduced. In the coupled model, the continuity of pressure and the mass balance at intersections of hydro-fractures are automatically achieved by sharing a common fluid node. The contact behavior of frictional fractures is modeled using the penalty method within the framework of plasticity theory of friction. Moreover, the extended Renshaw and Pollard criterion is utilized to predict whether a hydro-fracture will propagate across the frictional fracture. Simulation results indicate that the reduction technique can significantly accelerate the simulation without worsening the convergence or losing the computational accuracy for both types of iterations, and the acceleration effect becomes more remarkable as the problem scale increases. The great advantages of XFEM as well as the computational efficiency make the proposed method an attractive tool for engineering design of hydraulic fracturing treatments.

1. Introduction

Hydraulic fracturing has been used in a wide range of engineering areas. As an effective stimulation strategy, it is widely applied to enhance production of conventional and unconventional oil and gas reservoirs [1]. Other applications include underground disposal of toxic wastes [2], stimulation of geothermal reservoir [3] and secure storage of CO_2 [4]. A typical hydraulic fracturing process involves using a high-pressure fluid to pressurize the wellbore until fractures emerge, which

* Corresponding author.
E-mail address: wuha@ustc.edu.cn (H. Wu).

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Nomenclature

a	vector of enriched DOF associated with Heaviside function
b	vector of enriched DOF associated with tip enrichment function
B	matrix of shape function derivatives
c	vector of enriched DOF associated with junction enrichment function
D	elastic tensor
$\mathbf{D}^{\mathrm{cont}}$	contact tangent operator
E	elastic modulus
F_l	tip enrichment functions
$\mathbf{F}$, $\mathbf{F}^{\mathrm{ext}}$	force vectors
$\mathbf{F}_s$, $\mathbf{F}_e$	force vectors extracted from **F**
H	Heaviside function
H	global flux stiffness of fluid elements
$\mathbf{J}$, $\mathbf{J}_R$	Jacobian matrix of Newton-Raphson iteration
J	junction enrichment function
k	permeability of fracture
k_N, k_T	penalty parameters
K_{I}, K_{II}	mode-I and mode-II stress intensity factors
K_{e}	equivalent stress intensity factor
K_{IC}	fracture toughness
K_{m}	dimensionless fracture toughness
K	global stiffness matrix
$\mathbf{K}_{ss}$, $\mathbf{K}_{se}$, $\mathbf{K}_{es}$, $\mathbf{K}_{ee}$	sub-matrices extracted from **K**
$\mathbf{n}$, $\mathbf{n}_{\Gamma}$	outwards normal vectors
N	standard finite element shape function
N^p	shape function of fluid element
$\mathbf{N}^p$, $\mathbf{N}^u$, $\mathbf{N}^w$	matrices of shape functions
p	fluid pressure
P	fluid pressure vector
q	fluid flux
Q_{inj}	injection rate of fluid
Q	matrix transferring fluid pressure into equivalent nodal forces
$\mathbf{R}$, $\mathbf{R}_R$	residual vectors of Newton-Raphson iteration
s	coordinate system along hydraulic fracture
s_{tip}	location of fracture tip
S	sets of nodes
S	source term in coupled equations
t, Δt	time and time increment
$\mathbf{t}$, $\mathbf{t}^{\mathrm{cont}}$	traction vectors
$\mathbf{U}$, $\mathbf{U}_s$, $\mathbf{U}_e$	global nodal displacement
w	fracture width
w	fracture width vector
α	fracture propagation angle in the local fracture tip coordinate system
$\boldsymbol{\sigma}$	stress tensor
$\boldsymbol{\varepsilon}$	strain tensor
ε^w_{tol}, ε^p_{tol}, ε^c_{tol}	convergence tolerances
η_p, η_w, η_c	convergence factors
ν	Poisson's ratio
μ	fluid viscosity
μ_f	coulomb friction coefficient
$\boldsymbol{\Omega}$	two dimensional domain
Γ, Γ_{FF}, Γ_{HF}, Γ_t, Γ_u	boundaries

is followed by continuous injection of a large amount of fluid into emerged fractures to drive them to extend farther into the formation. In oil and gas fields, microseismic monitoring and other techniques have shown that pre-existing natural fractures in the formation further complicate the hydraulic fracture, forming complex fracture network [5–7]. In addition, the short- and long-term production of the reservoir is directly related to the complexity of the created fracture network which