ARTICLE IN PRESS

Engineering Fracture Mechanics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Ordinary state-based peridynamics for thermoviscoelastic deformation

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ARTICLE INFO

Article history:
Received 21 October 2016
Received in revised form 14 February 2017
Accepted 15 February 2017
Available online xxxx

Keywords:
Peridynamics
Viscoelasticity
Isotropic
Prony series
Failure

ABSTRACT

This study presents the ordinary state-based peridynamic (PD) constitutive relations for viscoelastic deformation under mechanical and thermal loads. The behavior of the viscous material is modeled in terms of Prony series. The constitutive constants are the same as those of the classical history-integral model, and they are also readily available from relaxation tests. The state variables are conjugate to the PD elastic stretch measures; hence, they are consistent with the kinematic assumptions of the elastic deformation. The PD viscoelastic deformation analysis successfully captures the relaxation behavior of the material. The numerical results concern first the verification problems, and subsequently, a double-lap joint with a viscoelastic adhesive where failure nucleates and grows.

1. Introduction

Many structural materials exhibit viscoelastic behavior under high temperature applications. Polymers and solid-propellant materials are usually modeled as linear viscoelastic materials. Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Its response depends on loading rate, deformation history and its rate. Therefore, critical structures made of viscoelastic materials warrant the search for a better understanding of their mechanical behavior, including failure characteristics. Their time dependent behavior modeled by using Prony series is well understood from a computational point of view within the Classical Continuum Mechanics (CCM), and commercially available finite element programs enable the prediction of their response without any difficulty.

However, the CCM faces conceptual and mathematical challenges when addressing crack nucleation and growth especially in the presence of multiple crack paths. These challenges stem from the fact that the equations of CCM involve spatial derivatives of the displacement components, and do not contain an internal length scale. Peridynamics (PD), a reformulation of the CCM by Silling [11] and Silling et al. [13] removes these challenges by introducing an internal length scale, and integro-differential equations as opposed to the partial differential equations of CCM. It is extremely suitable for failure analysis of structures because it allows cracks to nucleate and grow naturally. An extensive literature survey on PD is given by Madenci and Oterkus [7].

The PD theory is not limited to elastic material response. Mitchell [9] presented the ordinary state-based PD viscoelastic model, and studied the relaxation response of a single bond in the case of a two parameter PD standard linear solid model. Also, Weckner and Mohamed [16] developed a bond-based PD micromodulus with viscoelastic effects. By considering an

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http://dx.doi.org/10.1016/j.engfracmech.2017.02.011
0013-7944/

Please cite this article in press as: Madenci E, Oterkus S. Ordinary state-based peridynamics for thermoviscoelastic deformation. Engng Fract Mech (2017), http://dx.doi.org/10.1016/j.engfracmech.2017.02.011

infinite viscoelastic bar subjected to a dynamic point load, they showed that the PD theory recovers the local viscoelastic response in the limit as the PD internal length approaches zero.

This study may be considered as an extension of Mitchell's pioneering study that employs the Strain Energy Density (SED) function suggested by Silling et al. [13] for the "*linear PD solid*". The main contribution concerns the use Prony series for modeling the behavior of viscous material for three- and two-dimensional PD thermoviscoelastic analyses. Also, it employs the PD form of the SED function suggested by Madenci and Oterkus [7] for a linear material response identical to that of classical continuum mechanics. It is expressed in terms of three PD material parameters specifically for small deformation and small rotations. These PD parameters for both three- and two-dimensional analyses are determined by calibration against the classical SED by considering the two simple loading conditions of isotropic expansion and simple shear.

However, this SED function employs a specific form of the weight function suggested by Madenci and Oterkus [7] because it recovers the bond-based PD formulation.

After performing its verification by comparison with finite element simulations, a double-lap bonded joint is considered to demonstrate the PD predictive capability for crack nucleation and growth. Although a uniform temperature distribution is considered as part of the verification process, this study is also capable of including non-uniform temperature variations.

2. Peridynamic equation of motion

The generalized PD equation of motion is a nonlinear integro-differential equation in time and space in the form [13]

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{H_{\mathbf{x}}} (\mathbf{t}(\mathbf{u}'-\mathbf{u},\mathbf{x}'-\mathbf{x},t) - \mathbf{t}'(\mathbf{u}-\mathbf{u}',\mathbf{x}-\mathbf{x}',t))dH_{\mathbf{x}} + \mathbf{b}(\mathbf{x},t), \tag{1}$$

where the region H defining the range of material point $\mathbf{x}$ is specified by δ, referred to as the "horizon." This equation can also be derived based on the principle of virtual work by satisfying the Lagrange's equation [7]. As shown by Silling and Lehoucq [14], the classical theory of elasticity can be considered a limiting case of the PD theory as the horizon approaches zero in Eq. (1). Since the integrand in Eq. (1) does not contain any spatial derivatives, it is valid everywhere regardless of the presence of discontinuities. This equation is not usually amenable for analytical solutions. Therefore, its solution is constructed by using numerical techniques for spatial and time integrations. The spatial integration is performed by using the collocation method of a meshless scheme due to its simplicity. Hence, this equation is discretized by replacing the integration with an infinite summation as

$$\rho_{(k)}\ddot{\mathbf{u}}_{(k)} = \sum_{j=1}^{\infty}\left[\mathbf{t}_{(k)(j)}(\mathbf{u}_{(j)}-\mathbf{u}_{(k)},\mathbf{x}_{(j)}-\mathbf{x}_{(k)},t) - \mathbf{t}_{(j)(k)}(\mathbf{u}_{(k)}-\mathbf{u}_{(j)},\mathbf{x}_{(k)}-\mathbf{x}_{(j)},t)\right]V_{(j)} + \mathbf{b}_{(k)} \tag{2}$$

However, the numerical integration is performed by dividing the domain into a finite number of specific volumes associated with the integration (material) points, N. The volume of each point is assumed equal to each other. Therefore, this equation is rewritten to reflect the range of summation, N which has a fixed value representing the number of material points in the horizon of material point, $\mathbf{x}_{(k)}$ as

$$\rho_{(k)}\ddot{\mathbf{u}}_{(k)} = \sum_{j=1}^{N}\left[\mathbf{t}_{(k)(j)}(\mathbf{u}_{(j)}-\mathbf{u}_{(k)},\mathbf{x}_{(j)}-\mathbf{x}_{(k)},t) - \mathbf{t}_{(j)(k)}(\mathbf{u}_{(k)}-\mathbf{u}_{(j)},\mathbf{x}_{(k)}-\mathbf{x}_{(j)},t)\right]V_{(j)} + \mathbf{b}_{(k)} \tag{3}$$

where the displacement $\mathbf{u}_{(k)}$ is the displacement vector, and $\mathbf{b}_{(k)}$ is the body load vector at the material point $\mathbf{x}_{(k)}$. The vectors $\mathbf{t}_{(k)(j)}$ and $\mathbf{t}_{(j)(k)}$ represent the force density acting on material points $\mathbf{x}_{(k)}$ and $\mathbf{x}_{(j)}$, respectively. These force density vectors are unequal in magnitude with opposite directions. The family of material point $\mathbf{x}_{(k)}$ and $\mathbf{x}_{(j)}$ are denoted by $H_{\mathbf{x}_{(k)}}$ and $H_{\mathbf{x}_{(j)}}$, respectively, as shown in Fig. 1 As illustrated in Fig. 2, the material point $\mathbf{x}_{(k)}$ interacts with its family of material points, $H_{\mathbf{x}_{(k)}}$, and it is influenced by the collective deformation of all these material points. Similarly, material point $\mathbf{x}_{(j)}$ is influenced by deformation of the material points, $H_{\mathbf{x}_{(j)}}$, in its own family. The number of family members in each family remains the same, and their motion conforms to the Lagrangian.

As proposed by Silling [11], the horizon can be determined by matching the PD dispersion curves with those measured for a specific material. Weckner and Silling [17] matched dispersion curves from the PD and atomistic simulations. However, such a horizon was on the order of the atomic spacing. Therefore, the PD horizon can be viewed as an "*effective interaction distance*" or an "*effective length-scale*" of a continuum model, and it may depend on the material properties, geometry and loading.

The horizon is related to the grid size. Therefore, Tian and Du [15] developed Asymptotically Compatible (AC) discretization schemes for robust approximations of PD models and their local continuum models. AC schemes allow for the preservation of the consistency between nonlocal and local limits of the continuum model at the discrete level, regardless of how the grid spacing between the material points is compared with the horizon.

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