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The surface-forming energy release rate versus the local energy release rate

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ABSTRACT

This paper identifies two ways to extract the energy (or power) flowing into a crack tip during propagation based on the power balance of areas enclosed by a stationary contour and a comoving contour. It is very interesting to find a contradiction that two corresponding energy release rates (ERRs), a surface-forming ERR and a local ERR, are different when stress singularity exists at a crack tip. Besides a rigorous mathematical interpretation, we deduce that the stress singularity leads to an accompanying kinetic energy at the crack tip. The local ERR G_L represents the driving force to overcome the surface energy and the accompanying kinetic energy, while the surface-forming ERR G_s represents the driving force to overcome the surface energy only. Their advantages and disadvantages are discussed. We recommend using the surface-forming ERR G_s based fracture criterion for a crack propagation in elastic-plastic materials, since it has a wide applicability and concise formulae which are easy to compute among all energy based criteria.

1. Introduction

For elastic-plastic fracture problems, the *J*-integral [1] based fracture criterion is widely used when there is no crack propagation. However, if a crack propagates, the plastic unloading will appear and then the strain energy density in the *J*-integral cannot be defined unambiguously. Many researchers tried to put forward different methods to solve this problem [2–8]. For example, Brust and his collaborators [9–11] defined a path-independent integral, i.e. T^* integral, by introducing the total accumulated increments of stress working density for an incremental plasticity theory. They further found that the *J*-integral and T^* curves were almost coincident for a small amount of crack growth, but deviated from each other as the crack further grows. When the crack growth reaches to steady state, the *J*-integral unreasonably continues to rise, while the T^* turns to be a constant. By invoking the second law of thermodynamics, Simha et al. [12] derived the near-tip and far-field *J*-integrals for a growing crack in finite deformation regime with incremental plasticity. Definition of the potential or the stored energy density in their paper is not clear enough when plastic unloading appears. The advanced framework of the configurational force and the complex plastic constitutive relation in their paper may influence its acceptance to mechanicians. Although these works are very important steps to understand or solve the fracture problems of elastic-plastic materials, to the best of our knowledge, they have not been widely used yet.

We note that the energy definition is usually controversial and inconsistent among many criteria and leads to confusion for users, such as the stress working density used by Brust [9–11], but Helmholtz free energy used by Simha et al. [12].

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Therefore, adopting the power balance to avoid any energy definition should be a better starting point to study the elastic-plastic crack propagation problems. In our previous paper [13], a surface-forming energy release rate G_s is defined based on the power balance, which represents the energy available for separating the crack surfaces during the crack propagation and excludes the loading-mode-dependent plastic dissipation. We also proposed the corresponding fracture criterion, which has no limitation on the constitutive behaviors of materials and has a wider applicability. Moreover, a reasonable interpretation of Rice paradox on crack propagation in elastic-perfectly plastic materials was given.

However, an interesting contradiction in that paper was pointed out to us by Prof. Landis from the University of Texas at Austin. We find that our further investigation on this contradiction can disclose a derivation error, which has been ignored not only in our previous paper [13], but also in some textbooks and lecture notes [14,15]. This investigation thus can deepen our understanding on the fracture mechanisms and behaviors.

The paper is structured as follows. In Section 2, we will introduce two energy release rates based on the power balance of the area within a contour, and then point out an interesting contradiction between them. Concise formulae of the two energy release rates are derived first and their physical meanings are illustrated and compared in Section 3. In Section 4, we discuss several issues on determining and simulating crack propagation in elastic-plastic materials. The conclusions are summarized in Section 5.

2. An interesting contradiction between two energy release rates

As the energy cannot be defined unambiguously in elastic-plastic materials, the energy release rate can be introduced through the power balance during the crack propagation. There are two ways to establish the power balance relations. The first one is to investigate the power balance within a fixed contour Γ surrounding the crack tip as shown in Fig. 1. x_1, x_2 is a stationary coordinate system (fixed on the material points), A is the area enclosed by the contour, and $\mathbf{n}$ is the unit external normal vector. The power balance during a crack propagation can be written as

$$G_s\dot{a} = \int_\Gamma n_j\sigma_{ij}\dot{u}_i d\Gamma - \int_A \sigma_{ij}\dot{\varepsilon}_{ij} dA \tag{1}$$

where G_s represents the power available for separating the crack surfaces and is named as the surface-forming energy release rate (ERR). a is the crack length, σ_{ij}, ε_{ij} and u_i are stress, strain and displacement components, respectively. $(\dot{\bullet})$ represents the temporal derivative $\left.\frac{\partial()}{\partial t}\right|_{x_1,x_2}$. The term $\int_\Gamma n_j\sigma_{ij}\dot{u}_i d\Gamma$ is the power of the external force, and $\int_A \sigma_{ij}\dot{\varepsilon}_{ij} dA$ is the power of the internal force, including the rate of the stored elastic strain energy $\int_A \sigma_{ij}\dot{\varepsilon}^e_{ij} dA$ and the rate of energy dissipation by plastic deformation $\int_A \sigma_{ij}\dot{\varepsilon}^p_{ij} dA$ in surrounding material points.

The power balance relation, Eq. (1), is simple, clear and correct, and the physical meaning of the surface-forming ERR G_s is also explicit. Obviously, G_s based criterion is suitable for both elastic-plastic crack propagation and other situations. However, it is very interesting to find that for steady-state crack propagation in linear elastic materials, G_s cannot degenerate to the J-integral, and the detailed derivation is presented in Appendix A. This inconsistence between the surface-forming ERR and J-integral stimulates us to investigate the power balance in the second way as well.

We study the area within a comoving contour Γ surrounding the crack tip as shown in Fig. 2. x'_1, x'_2 is a comoving coordinate system (moving with the crack tip), and A_{mov} is enclosed by the moving contour. At the initial moment t_n, two coordinate systems (x'_1, x'_2 and x_1, x_2) coincide and the crack length is denoted by a_n. At a later moment t, we have the relationship

$$x'_1 = x_1 - [a(t) - a_n], x'_2 = x_2 \tag{2}$$

where $a(t)$ is the corresponding crack length.

The corresponding power balance relation during crack propagation can be written as

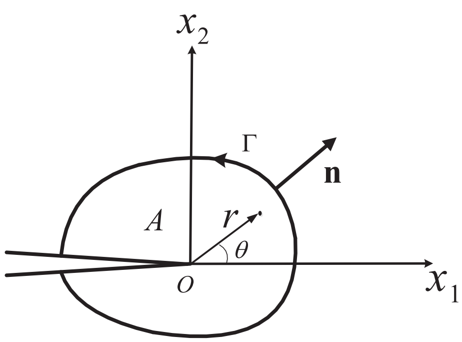

Fig. 1. Schematic of a stationary contour surrounding the crack tip.