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Comparison of boundary and size effect models based on new developments

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ABSTRACT

Hoover, Bazant and colleagues have published a number of papers in recent years (Bazant and Yu, 2009; Yu, 2010; Hoover and Bazant, 2013a, 2013b, 2014) on comparisons between Bazant size effect model (SEM) and Hu-Duan boundary effect model (BEM) for quasi-brittle fracture of concrete. With the recent developments of BEM (Wang et al., 2016; Guan et al., 2016; Wang and Hu, 2017) on irregular and discrete crack growth in concrete shaped by coarse aggregate structures, it is time to clarify issues on the SEM and BEM comparison raised by Bazant and Yu (2009), Yu (2010), Hoover and Bazant (2013a, 2013b, 2014). The experimental results of Hoover and Bazant (2013a, 2013b, 2014) are analyzed again using BEM, and new findings and in-depth understandings that have not been achieved by SEM are presented in this study. BEM is one concise equation, containing only two fundamental material constants, tensile strength f_t and fracture toughness K_{IC}, applicable to both notched and un-notched concrete specimens. Most importantly, BEM explains the inevitable influence of coarse aggregate structures on quasi-brittle fracture of concrete through modeling irregular and discrete crack formations and by considering the critical role of the maximum aggregate $d_{\max}$. In contrast, SEM has three different equations, one for notched, one for un-notched, and one for shallow-notch specimens, containing total 18 empirical parameters to be determined from curve fitting. Despite with the staggering 18 parameters, the three SEM equations still overlook the crucial role of coarse aggregate structures in concrete fracture; $d_{\max}$ and discrete crack formation are not considered. After establishing the relation between discrete fictitious crack formation Δa_{fic} and $d_{\max}$ at the peak load $P_{\max}$ based on four different sets of independently obtained experimental results of concrete and rock with $d_{\max}$ from 2 to 10 and 19 mm, BEM becomes a predictive design model which only needs strength f_t and toughness K_{IC}.

1. Introduction

Quasi-brittle fracture of concrete refers to irregular and intermittent micro-crack formations at a notch tip within the coarse aggregate structures of a concrete specimen before the maximum fracture load $P_{\max}$ is reached. The commonly assumed straight-line fictitious crack growth Δa_{fic} at the peak load $P_{\max}$ should reflect the intermittent and discrete crack

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Nomenclature

a_0	initial notch length for notched specimens
a_{eff}	effective notch length for un-notched specimens (>0 due to coarse structures)
a_e	equivalent notch length linked to a_0 and specimen boundary and size conditions
B_T	thickness of specimen
$d_{\max}$	maximum aggregate size
f_t	tensile strength
K_{IC}	fracture toughness – (for large concrete structure with long notch >> aggregate)
$a* \infty$	characteristic crack length – material constant fully determined by f_t and K_{IC}
$P_{\max}$	maximum applied load at fracture
S	span of specimen between supports
W	width of specimen
$Y(\alpha)$	geometry factor in the stress intensity formula
α	α-ratio = a_0/W
Δa_{fic}	fictitious crack growth ahead of the initial notch at $P_{\max}$
β	discrete number for fictitious crack growth at $P_{\max}$ ($\Delta a_{fic}/d_{\max}$ ratio)
σ_n	nominal strength at the notch plane for Boundary Effect Model (BEM)
σ_N	nominal strength without consideration of notch for Size Effect Model (SEM)
σ_{bri}	crack bridging stress over fictitious crack surface
FPZ	Fracture Process Zone or the length of a fictitious crack with crack bridging stress
BZ	Boundary Zone of specimen or structure for quasi-brittle fracture

growth mimicking the coarse aggregate structures. Continuous crack growth and smooth crack-bridging stress distributions over Δa_{fic}, commonly adopted by continuum mechanics models are not the most convincing explanations of quasi-brittle fracture in concrete with coarse aggregate structures. Such modeling has practically ignored the heterogeneity of coarse aggregate structures or has equivalently assumed concrete is homogeneous in modeling.

The assumed straight-line Δa_{fic} at $P_{\max}$ is difficult to measure but longer fracture process zone (FPZ) in concrete after $P_{\max}$ has been measured successfully using acoustic emission, e.g. by Otsuka and Date [9], Ohno [10], Ohno et al. [11], Muralidhara et al. [12], and X-ray technique by Otsuka and Date [9] and Kumpova et al. [13]. The 3D images from acoustic emission and X-ray measurements show that FPZ or Δa_{fic} after $P_{\max}$ is strongly influenced by the maximum aggregate size $d_{\max}$, and micro-crack formations in front of the notch a_0 are highly irregular and discontinuous. It can be envisaged that because of the coarse aggregate structures and associated heterogeneous properties, such as weak planes/sites and defects around aggregates and aggregate distributions and locations, discrete or stepwise crack growth in concrete is expected.

To truly understand the quasi-brittle fracture process in concrete and the associated irregular and intermittent micro-crack growth within the coarse aggregate structures, the maximum aggregate size $d_{\max}$, or alternatively the average aggregate size for a concrete mix, has to be included in analytical modeling, particularly if the specimen size W and $d_{\max}$ ratio is only around 20 or less ($W/d_{\max} < 20$). Unfortunately, this has not been the case although size effect on quasi-brittle fracture of concrete has been studied for over 30 years since the original size effect study of Bazant in 1984 [14]. Even in recent years, when Hoover and Bazant [1–5] compared Bazant SEM with Hu-Duan BEM, the crucial function of coarse aggregate structures or $d_{\max}$ was still not considered. A continuum mechanics approach on quasi-brittle fracture of concrete, ignoring the inevitable influence of coarse aggregate structures even under the condition $W/d_{\max} < 20$, is resting on a shaky foundation regardless whether it is SEM or BEM.

As summarized previously by Karihaloo et al. [15] and more recently by Caglar and Sener [16], the main functions of commonly accepted size effect models on quasi-brittle fracture of concrete are typically limited to curve fitting of experimental results obtained from geometrically similar specimens with a constant notch/specimen-size ratio, or $\alpha = a_0/W =$ constant. Those models are not predictive design equations even if the fundamental material properties such as tensile strength f_t and fracture toughness K_{IC} are given for a concrete mix. Most critically, the maximum aggregate $d_{\max}$ and inevitable irregular and discontinuous micro-crack formation within the coarse aggregate structures have not received the due attention they deserve.

To break away from the 30-year old tradition of empirical curve fitting and to develop true predictive design models for concrete fracture, researchers particularly young researchers should be encouraged to question the existing fracture models, including SEM and BEM, if obvious faults exist. One obvious question is why the existing size effect models on quasi-brittle fracture of concrete do not consider the coarse aggregate structures or $d_{\max}$, while the coarse aggregate structures are known to be the fundamental source of quasi-brittle fracture and size effect?

Instead of blindly following those existing size effect models, researchers should pay close attention to the paper by Karihaloo et al. [15], which has shown different size effect models can be used to describe the same set of experimental data if empirical parameters can be adjusted freely. In a recent publication by Caglar and Sener [16], it has been shown that experimental results from un-notched specimens can be fitted equally well by Bazant SEM (for un-notched specimens only) and