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An improved definition for mode I and mode II crack problems

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ABSTRACT

In this research, the common definitions for mode I and mode II are evaluated and improved. For this purpose, the in-plane linear elastic stress field around the crack tip is written as a set of infinite series expansions. Mode I and mode II fields are classically defined as symmetric and anti-symmetric parts of these expansions, respectively. There is also a constant term called "*T*-stress" in these expansions; parallel to the crack line and independent of the distance from the crack tip. Previous definitions assume that *T*-stress exists only in pure mode I or combined mode I and mode II conditions. Based on these definitions, *T*-stress always vanishes in pure mode II. However, the published results of several analytical and experimental researches indicate that the constant stress term can exist in mode II stress field, as well. In this paper, some examples are presented which indicate the presence and importance of *T*-stress in pure mode II conditions. Then, the classical definition for mode I and mode II is modified to make it consistent with the results presented in the literature.

1. Introduction

Cracks are generated in many engineering structures and components during their service lives. These cracks influence the stress distribution in the component and can result in significant reduction in its strength and life. Because of the paramount importance of safety in engineering components, the crack problem has been of interest to a large number of researchers.

The elastic stress field around the crack tip can be written as an infinite series expansion [1]. The in-plane stress components near the crack tip are usually expressed as:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{-\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right] + T + O(r^{1/2}) \quad (1.a)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + O(r^{1/2}) \quad (1.b)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + O(r^{1/2}) \quad (1.c)$$

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Nomenclature

a	crack length
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
K^*	normalized stress intensity factor
K_{IIc}/K_{Ic}	fracture toughness ratio
P	concentrated force
r	distance from the crack tip
R	radius
$2S$	distance between supports
T	T-stress term
T^*	dimensionless form of T-stress
α	crack angle
$\chi(r,\theta)$	airy stress function
λ	load biaxiality ratio
θ	polar coordinate, angle
σ	stress

where r and θ are the polar coordinates centered at the crack tip (Fig. 1). K_I, K_{II} and T are crack parameters, which have an important role in brittle fracture of cracked components.

In general, two main independent modes are observed in a cracked body under in-plane loading conditions. Mode I is opening mode in which the crack faces tend to separate in direction normal to the crack line; and Mode II is shearing mode, associated with in-plane sliding of crack faces over each other. This descriptive definition is valuable and useful for acknowledging the deformation modes of crack. However, as elaborated in this paper, the definition is not enough to quantify the fracture parameters in engineering problems.

In the field of the fracture mechanics, a great deal of study has been devoted to develop criteria for brittle fracture. These criteria are often used for cracks under combined modes I and II loading conditions. Many different criteria such as maximum strain energy release rate (MERR) [2,3], maximum tangential stress (MTS) [4], maximum tangential strain (MTSN) [5], maximum tangential strain energy density (MTSE) [6], and minimum strain energy density (SED) [7,8] have been suggested and used in literatures. These criteria generally use a mechanical parameter (e.g. stress, strain, strain energy) and its critical value to predict the crack growth initiation through some equations or diagrams. For utilizing the mixed made fracture criteria, elastic stress and strain fields in close vicinity of the crack tip must be determined. These fields are not the same for different modes of crack deformation, making it necessary to recognize the crack modes and their contribution in mixed mode problems. Hence, an accurate definition for these modes is important in the interpretation of the results in mixed mode I/II crack problems.

In this study, the common definitions for crack modes are reviewed. It is taken into consideration that the previous definitions always neglect the T-stress term in mode II condition. However, a review of literature shows that sometimes there is very poor agreement between theoretical and experimental results in mode II fracture tests [e.g. 4,9]. Some researchers have

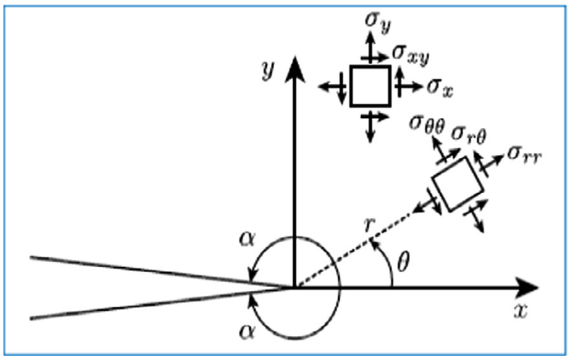

Fig. 1. Crack tip coordinates and stress components.

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