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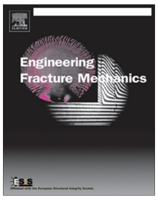

Normalization of cohesive laws for quasi-brittle materials

Johan Tryding [*], Matti Ristinmaa

Division of Solid Mechanics, Lund University, SE-221 00 Lund, Sweden

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Analytical relations to describe experimentally measured traction-separation laws are often expressed in dimensionless quantities. The traction and the separation are commonly normalized using the cohesive strength and a length measure, respectively. The ratio between the fracture energy and the cohesive strength is often used as a length measure. An alternative length measure is the ratio between the cohesive strength and the maximum slope of the traction-separation law. A relation between these two length measures are established. To illustrate the implications on cohesive laws, three existing cohesive laws are rewritten using the alternative normalization. As a result it is shown that the number of unknown material parameters can be reduced. One of the derived dimensionless cohesive law is validated against experimental uniaxial tension and compression load-deformation data of different sample sizes and different quasi-brittle materials, i.e. concrete and paperboard. A good fit of the cohesive law is shown to all the investigated data. These findings indicate that the derived normalized cohesive law is independent of material directions, moisture contents and sample size.

1. Introduction

Cohesive crack models utilizing traction-separation laws are successfully applied to a wide range of materials, such as concrete and rocks [1], paper based materials [2], adhesive joints [3] and composite materials [4], to mention a few. The Hillerborg cohesive concept [1] has also been implemented into several commercial general purpose finite element codes. The usage of the cohesive zone concept in industrial applications, is illustrated e.g. in [5–7]. Thus knowing the traction-separation laws for the quasi-brittle material studied, simulation can be used to predict cohesive fracture.

For arbitrary loading histories the cohesive law needs to include the characteristics of both the normal and shear cohesive behaviors, cf. [4,8]. Recently in [9,10] a thermodynamically consistent cohesive crack model for generally normal and shear loading was proposed utilizing only a calibration of the traction-separation law for tension loading. The model includes the Helmholtz free energy, loading and plastic potential functions together with a damage potential function calibrated to an analytical cohesive law in uniaxial tension.

Analytical traction-separation laws used to model, e.g. concrete data, range from linear, bilinear, exponential to non-linear functions, cf. [1,11–14]. These functions are often expressed as dimensionless analytical cohesive laws, cf. e.g. [5], where the cohesive stress commonly is normalized using the cohesive strength. The normalization of the separation is not as obvious as for the cohesive traction and a frequently used normalization is obtain by relating the separation to the ratio between the fracture energy and the cohesive strength, cf. e.g. [5]. An alternative length measure is found from the ratio

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[*] Corresponding author.

E-mail address: johan.tryding@tetrapak.com (J. Tryding).

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Nomenclature

c	dimensionless parameter
$N_{\max}$	cohesive maximum slope
T	cohesive stress
T_0	cohesive strength
T_c	cohesive strength at uniaxial compression
x_c	dimensionless parameter
Γ_0	cohesive fracture energy
Γ_c	cohesive fracture energy at uniaxial compression
δ	separation
δ_0	critical separation when the stress is zero
$\delta_c r$	critical interpenetration when the crushing zone is able to transfer a constant stress
δ_G	normalized separation length
δ_N	normalized separation length

between the cohesive strength and the maximum slope of the cohesive law, as suggested in [15]. The length measure was identified from a dimensionless analytical cohesive traction-separation law for diatomic molecules, utilizing the Morse potential [16].

The two material parameters cohesive strength and fracture energy are today well known material parameters [1], however, this is not the case for the maximum slope. The maximum slope is an important material property and used to establish conditions for stable uniaxial tensile testing. The ratio of the maximum slope and the elastic modulus provides a measure of the sample length needed to obtain a experimental test setup giving both the ascending and the descending force-displacement relation [11].

In this work the normalization proposed in [15] of the cohesive law is applied to experimental data and for this purpose an analytical cohesive law proposed for concrete in [14] will be investigated. In Section 2 the two normalizations of the separation and analytical traction-separation laws are discussed. In Section 3 experimental data from uniaxially loaded concrete and paperboard materials, existing in the literature as well as supplemented with new data, and a method to calibrate the data to a specific softening function, are discussed. The data set contain samples having different sizes, geometries and moisture contents. In Section 4 the two normalizations of the separation are analyzed and the investigated analytical cohesive law are calibrated to the experimental data. In Section 5 the results are discussed and conclusions are made. In the text we will adopt the guidelines for cohesive models nomenclature defined in [17].

2. Theory

2.1. Normalization of traction-separation laws

After the peak load and at additional displacements the deformation in the loaded specimen is no longer uniform. A localized cohesive zone is formed, experimentally observed as a localized narrow band. As the elongation of the specimen continues, the localized cohesive zone widens and the stress decreases. For tensile loading the opening of the cohesive zone defines the separation, δ, of the cohesive zone.

Experimental relations between the cohesive stress, T, and the separation, δ, reported in the literature, cf. [18,5], are usually of the format

$$\frac{T(\delta)}{T_0} = \hat{f}\left(\frac{\delta}{\delta_G}\right), \tag{1}$$

where T_0 is the cohesive strength, δ_G is a normalizing separation length and $\hat{f}(\delta/\delta_G)$ is a continuously or piece-wise differentiable monotonic decreasing function. Hence, the cohesive stress in Eq. (1) is scaled with the cohesive strength, whereas the separation is scaled with a normalizing separation length measure. The normalization of the separation is not as obvious as for the cohesive stress. A frequently used normalization is obtained by relating the length measure δ_G to the fracture energy Γ_0 and the cohesive strength T_0 as, cf. [18,5],

$$\delta_G = \frac{\Gamma_0}{T_0}. \tag{2}$$

As shown in [15], using the Morse potential for diatomic molecules, a traction-separation law can be identified as

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