Engineering Fracture Mechanics 171 (2017) 35–49

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Stress intensity factor solutions for several crack problems using the proportional crack opening displacements

Xin Lan [a,*], Shaobo Ji [a], Nao-Aki Noda [b], Yong Cheng [a]

[a] School of Energy and Power Engineering, Shandong University, Jingshi Road 17923#, Jinan, Shandong Province, China
[b] Department of Mechanical Engineering, Kyushu Institute of Technology, 1-1 Sensui-cho, Tobata-ku, Kitakyushu-shi, Fukuoka 804-8550, Japan

ARTICLE INFO

Article history:
Received 25 May 2016
Received in revised form 3 December 2016
Accepted 7 December 2016
Available online 24 December 2016

Keywords:
Stress intensity factor
Crack opening displacement
Interfacial crack
Finite element method

ABSTRACT

A general finite element procedure based on the proportional crack opening displacements for obtaining the stress intensity factors is presented. The procedure is applied to the non-singular 3-node linear, 4-node linear, 8-node parabolic, 8-node axisymmetric elements and 8-node hexahedral solid elements for a test. It is found that the current method exhibits good element type adaptability and significantly less mesh dependency, and accurate results can be obtained effectively using rather coarse meshes. The accuracy of the current procedure is evaluated by applying it to two-dimensional interface cracks, three-dimensional penny-shaped cracks as well as circumferential surface cracks. Comparison with the published data from the literature shows that the current procedure gives accurate stress intensity factors. Furthermore, the current method is fairly efficient and less computational resource consuming and can be used as an effective tool in the reliability analysis of the bonded multi-layers.

1. Introduction

Bi-material interfaces are widely observed in the modern composite structures. The presence of an interface crack may eventually cause a through thickness crack which results in the final failure of a structure. The singular stress field around an interface crack was firstly discovered by Williams [1], then his work was followed and extended by Rice and Sih [2], Erdogan [3,4], England [5], Willis [6] and many others. Following their pioneering research, a variety of algorithms have been developed based on LEFM and in conjunction with the analytical method or the numerical method. The analytical methods for solving the stress intensity factors (SIFs) for the interfacial crack problems are only limited to a few specific cases due to the inherent mathematical difficulties. Therefore, general numerical methods are necessary to be employed to treat the more general cracked bodies in the practical applications. In this paper, a brief summary regarding the numerical methods available for computing the SIFs of the interface cracks using FE analysis will be reviewed and discussed. Then, a finite element procedure using the proportional relative crack opening displacement (COD) for obtaining the SIFs of the interfacial cracks will be proposed.

Just mention a few of those procedures using FE analysis, Matos et al. [7] proposed a numerical method using FE analysis to compute the SIFs of an interface crack. This method is based on the evaluation of the J-integral by the virtual crack extension method. Then, individual stress intensities were obtained from further calculations of J perturbed by small increments.

* Corresponding author.
E-mail address: lanxin@sdu.edu.cn (X. Lan).

http://dx.doi.org/10.1016/j.engfracmech.2016.12.002
0013-7944/

Nomenclature

Latin symbols

a	length of an edge interface crack or half-length of a central interface crack
e_{min}	minimum element size of the FE model
E_1, E_2	Young's moduli of material 1 and 2
F_I, F_{II}	normalized stress intensity factors of an edge interface crack
G	strain energy release rate
i	imaginary unit, equal to the square root of -1
K_I, K_{II}	general stress intensity factors of the given unknown problem
K_I^*, K_{II}^*	general stress intensity factors of the reference problem
K_I', K_{II}''	general stress intensity factors using a reference length $l = 2a_1$
r	polar distance away from the singular point/crack tip
S, T	remote shear and tension applied to the reference problem
u_x, u_y	nodal displacement components
W	width of the bonded strip

Greek symbols

δ_x, δ_y	relative crack opening displacement components
$\delta_{y,A1}^{T=1*}, \delta_{x,A1}^{T=1*}$	crack opening displacement of Problem A1 subjected to pure unit tension $T = 1$
$\delta_{y,A2}^{S=1*}, \delta_{x,A2}^{S=1*}$	crack opening displacement of Problem A2 subjected to pure unit shear $S = 1$
$\delta_{y,A}^*, \delta_{x,A}^*$	crack opening displacement of the reference problem (Problem A)
$\delta_{y,B}, \delta_{x,B}$	crack opening displacement of the given unknown problem (Problem B)
ε	bi-elastic constant
θ_1, θ_2	angles of the traction-free edges intersect the interface
κ_m	Kolosov constant
μ_m	shear modulus of material m
ν_m	Poisson's ratio of material m
$\sigma_{x1}^\infty, \sigma_{x2}^\infty$	remote transversal tension applied to the bonded half-planes

Sub/superscripts

m	m = 1, 2, material 1 or 2
A, B	the reference problem (problem A) and the given unknown problem (problem B).
I, II	mode I and II components
x, y	x direction and y direction of the coordinate at the crack tip
*	reference problem

Abbreviations

BEM	boundary element method
COD	crack opening displacement
FEM	finite element method
LEFM	linear elastic fracture mechanics
SIF	stress intensity factor

Chow and Atluri [8] got the SIFs of the interfacial cracks using the virtual crack closure integral with relatively coarse finite element meshes. In their procedure, the strain energy release rates should be computed in advance using the method proposed by Rybicki and Kanninen [9] as well as Raju [10]. Sun and Qian [11] used finite elements in conjunction with the crack closure method to obtain strain energy release rates [12] from which the SIFs could then be derived. The aforementioned procedures resorted to the use of the strain energy release rate to produce the final SIFs. Yuuki and Cho [13] determined the SIFs of the interface cracks by means of the extrapolation of the crack surface displacement. In this method, it needs skills to select the effective data area to determine the slope of the extrapolated line. Oda et al. [14] obtained the SIFs of the interface cracks using the ratios of the crack tip stresses. His concept was extended from the crack tip stress method proposed by Teranishi and Nisitani [15] for the homogeneous cracks. Noda and Lan [16] investigated the robustness of Oda's method and proposed a linear extrapolation technique to improve the accuracy. However, both the very refined meshes and the extrapolation technique add to the extra computational costs which lead to the lower efficiency.