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Modeling of multiple crack propagation in 2-D elastic solids by the fast multipole boundary element method

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ABSTRACT

In this paper, a fast multipole boundary element method (BEM) is presented for modeling crack propagation in two-dimensional (2-D) linear elastic solids. A dual boundary integral equation (BIE) formulation using a linear combination of the displacement and traction BIEs is applied to model cracks in this BEM. Constant boundary elements are used to discretize the BIEs and the fast multipole method (FMM) is applied to accelerate the solution of the BEM system of equations. Numerical examples of multiple crack propagation in 2-D elastic domains and under cyclic loading, including perforated plates with multiple holes and cracks, are presented to show the effectiveness and efficiency of the developed fast multipole BEM.

1. Introduction

The boundary element method (BEM) based on the boundary integral equation (BIE) for elasticity theory [1] has been applied to solve crack problems for more than three decades (see, e.g., some reviews in Refs. [2–9]). The multidomain BEM was first introduced to solve crack problems [10] using only the displacement (singular) BIE, in which a cracked body is divided into subdomains using artificial boundaries connecting the cracks. In the late of 1980s and early 1990s, the traction (hypersingular) BIE [11–17] was introduced and the displacement discontinuity (or jump) across the crack surfaces is used as the primary unknown variable to solve the crack problems based on the one crack surface model. This one crack surface formulation using the traction BIE only has been shown to be equivalent to the displacement discontinuity method (DDM) proposed by Crouch in 1976 [18], when constant elements are applied for both 2-D and 3-D cases [19–21]. Since then, various dual BIE formulations [22–28] using different combinations of the displacement and traction BIEs have been applied to solve crack problems in more general settings. The BEM has also been applied successfully in modeling interface cracks and cracks in functionally-graded materials (e.g., [29–33]). More comprehensive reviews of the BEM for modeling crack problems can be found in Refs. [5,6,9].

Although the BEM is accurate in solutions and efficient in meshing for solving crack problems, the computational efficiency had been a huge hurdle for the method for a long time, as the BEM system of equations is dense and nonsymmetrical. With a direct equation solver, both the computing time and memory storage are of at least $O(N^2)$ complexity (with N being the number of unknowns). To improve the computational efficiency, the fast multipole method (FMM) pioneered by Rokhlin and Greengard [34–36] has been introduced in the BEM to solve the crack problems. Many large-scale BEM models of mul-

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tiple cracks have been solved successfully using the fast multipole BEM in static and dynamic load cases [37–45], as well as in the modeling of crack propagations [46]. Detailed reviews and discussions of the fast multipole BEM can be found in Refs. [9,37,47,48].

In this work, a new fast multipole BEM for modeling large-scale 2-D linear elastic fracture mechanics problems is presented. A dual BIE formulation using a linear combination of the displacement and traction BIEs is applied on all boundaries of the problem domain, including surfaces of the cracks. This dual BIE formulation was originated by Burton and Miller [49] for solving exterior acoustic wave problems to remove the fictitious eigenfrequencies in the BIE solutions, which has also been found very effective in solving crack problems in the context of potential theory, acoustics and elastodynamics [26,27,50,51]. Due to the use of the hypersingular BIE which requires C^1 continuity of the field at the collocation point [52], higher-order elements or nonconforming elements have been applied in those works [26,27,50,51]. In this work, however, constant boundary elements are used to discretize the dual BIEs. Good numerical results have been obtained in determining the stress intensity factors (SIFs) using constant elements for 2-D crack problems [45]. This work will further show that the dual BIE formulation discretized with constant elements can also be applied successfully to model crack propagation problems with the fast multipole BEM. Numerical examples of propagation of multiple cracks in perforated plates with many holes are given in this paper to show the effectiveness and efficiency of the developed fast multipole BEM.

2. BIE formulation

We first review the direct BIE formulation for modeling crack problems under static loading. We start with the following direct displacement BIE (conventional BIE or CBIE) for a 2-D elastic body containing cracks [1]:

$$\frac{1}{2}\mathbf{u}(\mathbf{x}) = \int_S [\mathbf{U}(\mathbf{x},\mathbf{y})\mathbf{t}(\mathbf{y}) - \mathbf{T}(\mathbf{x},\mathbf{y})\mathbf{u}(\mathbf{y})]dS(\mathbf{y}), \quad \forall \mathbf{x} \in S, \tag{1}$$

where S is the entire boundary of the problem domain (including all crack surfaces and the outer boundary of the domain, if present); $\mathbf{x}$ and $\mathbf{y}$ are the source point and field point, respectively; $\mathbf{u}$ and $\mathbf{t}$ are the displacement and traction vector, respectively; $\mathbf{U}$ and $\mathbf{T}$ are 2×2 matrices from the displacement and traction kernels in the Kelvin's solution, respectively [37]. It is assumed that the surface is smooth at the source point $\mathbf{x}$. For completeness, we list the expressions for the two kernels ($\mathbf{U}$ and $\mathbf{T}$) in index notation for the plane strain condition [37]:

$$U_{ij}(\mathbf{x},\mathbf{y}) = \frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)\delta_{ij}\log\left(\frac{1}{r}\right) + r_{,i}r_{,j}\right], \tag{2}$$

$$T_{ij}(\mathbf{x},\mathbf{y}) = -\frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}] - (1-2\nu)(r_{,i}n_j - r_{,j}n_i)\right\}, \tag{3}$$

in which μ is the shear modulus, ν is Poisson's ratio, r is the distance between the source point $\mathbf{x}$ and field point $\mathbf{y}$, $(\ )_{,i} = \partial(\ )/\partial y_i$, δ_{ij} is the Kronecker δ symbol, and n_i is the direction cosine of the normal.

For a crack with two surfaces denoted as S^+ and S^- (Fig. 1), if we let S^- collapse onto S^+ to form a one surface model for the crack, the displacement BIE (1) collocated on the crack surface S^+ is reduced to the following [26,53]:

$$\frac{1}{2}\Sigma\mathbf{u}(\mathbf{x}) = \int_{S^+} [\mathbf{U}(\mathbf{x},\mathbf{y})\Sigma\mathbf{t}(\mathbf{y}) - \mathbf{T}(\mathbf{x},\mathbf{y})\Delta\mathbf{u}(\mathbf{y})]dS(\mathbf{y}), \quad \forall \mathbf{x} \in S^+, \tag{4}$$

where $\Delta\mathbf{u} = \mathbf{u}|_{S^+} - \mathbf{u}|_{S^-}$, $\Sigma\mathbf{u} = \mathbf{u}|_{S^+} + \mathbf{u}|_{S^-}$, and $\Sigma\mathbf{t} = \mathbf{t}|_{S^+} + \mathbf{t}|_{S^-}$. Additional terms from integrals on the outer boundary or other crack surfaces may appear on the right-hand side of the equation. Note that Eq. (4) is insufficient when it is applied alone to solve a crack problem, as it contains both the displacement sum and displacement discontinuity across the crack surfaces (There are two unknown functions). Therefore, the traction (hypersingular) BIE was introduced in late of 1980s in the BEM for modeling crack problems. The direct traction BIE (hypersingular BIE or HBIE) is [7,22,37]:

$$\frac{1}{2}\mathbf{t}(\mathbf{x}) = \int_S [\mathbf{K}(\mathbf{x},\mathbf{y})\mathbf{t}(\mathbf{y}) - \mathbf{H}(\mathbf{x},\mathbf{y})\mathbf{u}(\mathbf{y})]dS(\mathbf{y}), \quad \forall \mathbf{x} \in S, \tag{5}$$

where $\mathbf{K}$ and $\mathbf{H}$ are 2×2 matrices from the two new kernels in the Kelvin's solution [37]. The two kernels ($\mathbf{K}$ and $\mathbf{H}$) for the case of plane strain are given in the following [37]:

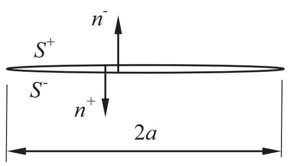

Fig. 1. A crack in a 2-D elastic domain (crack surface $S = S^+ \cup S^-$).