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Inclined circular punch with one or two ends in smooth contact with a cracked half plane

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ABSTRACT

An inclined circular punch with one or two ends in smooth contact (incomplete contact) with a half plane with a vertical edge crack is considered. The solution is analytically derived, and the conditions of smooth contact with the edge-cracked half plane are expressed as the restraining conditions. The inclined angle is related to the restraining condition at the smooth end of the punch. Because the restraining conditions belong to nonlinear equations, the problem cannot be simply solved using the method of superposition. Different inclined angles are assumed to iterate the corresponding stress intensity factors of the crack, resultant moment on the contact region and length of the contact region in different cases. The relations among the contact length, resultant moment about the origin, stress intensity factor and inclined angle of the punch are shown in the figures.

1. Introduction

The contact problem is an important problem in engineering and interesting in applied mechanics and mathematics. Many classical problems have been solved in the fine books [1–6], and many references were mentioned. The rigid-punch problem is also an important branch in contact mechanics, and the analysis of the interaction problem between rigid punch and crack (fracture mechanics) is useful and important. The interaction problem relates to the fretting fatigue problem [7]; therefore, many fretting problems have also been investigated.

Recently, the following punch problems of a half plane with a crack were studied by Panasyuk et al. [8,9], Babich et al. [10], Datsyshyn et al. [11,12], and Savruk and Tomczyk [13].

To better understand the present paper, we list the following rigid-punch problems of a half plane with a crack, which have been analyzed. A flat-ended rigid-punch problem of a half plane with an edge crack using a rational mapping function of the sum of fractional expression was analyzed in [14]. A wedge-shaped rigid punch was solved, which is also a solution for a flat-ended punch under rotation [15]. An edge-shaped rigid punch with one end in smooth contact (incomplete contact) with a cracked half plane was solved by satisfying the restraining condition at the smooth end [16]. An inclined circular rigid punch with two ends coming into a cracked half plane was solved by the superposition of two solutions: the solution of a circular rigid punch with two corners, which were vertically punched into a cracked half plane, and the solution of a

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Nomenclature

a	width of the punch
a'	contact length of the punch
b	crack length
c	coordinate of the crack position (Fig. 1(a) and (c))
m	index of Plemlj function (9)
$A_k, \overline{A_k}$	coefficients in stress function (8)
$E_0, E_k (k = 1, 2, \ldots, N), E_c$	coefficients of mapping function (3)
F_{Ia}, F_{IIb}	non-dimensional stress intensity factor of K_I, K_{II}
G	shear modulus
K_I, K_{II}	stress intensity factor
M_{ra}	non-dimensional resultant moment of R_m
N	number of fractional expressions of mapping function (3)
P	vertical force subjected to the punch
R	radius of curvature of the circular punch
R_m	resultant moment on the contact region about the origin
α, β	coordinates on the unit circle of the ends of the contact region
Δ	distance regarding position of vertical force P
ε	inclined angle of the punch
κ	function of ν (6)
μ	Coulomb's frictional coefficient
ν	Poisson's ratio
$\sigma_r, \sigma_\theta, \tau_{r\theta}$	stress components
ζ, σ	variables in and on the unit circle of the mapped plane
$\phi(\zeta), \psi(\zeta)$	stress functions on the mapped plane (8) and (11)
$\omega(\zeta)$	mapping function (3)
$\chi(\zeta)$	Plemelj function (9a)

flat-ended rigid punch under rotation [17]. A circular rigid punch that was vertically pushed into a cracked half plane with one end in smooth contact with the half plane was solved by satisfying the restraining condition at the smooth end [18,19]. A circular rigid punch that was vertically pushed into a cracked half plane with two smooth-contact ends was solved by simultaneously satisfying two restraining conditions at both ends of the rigid punch [20]. The inclination of the rigid punch was not considered in the previous papers [18–20] because the inclined angle is related to the restraining condition, and the method of superposition is no longer effective.

In the present paper, a general solution of the rigid-punch problem with an inclined angle is presented for one problem of the smooth contact at one end and another problem of two smooth contacts at two ends. The presence of a crack in the half plane and frictional force on the contact region usually result in the inclination of the punch. The restraining condition must be solved using a notably complicate iteration calculation. In particular, two simultaneous nonlinear equations for two smooth contacts must be solved using the iterative calculation. Therefore, some invention is necessary.

Although the rigid-punch problem can be calculated using numerical methods, e.g., the finite-element method, some inconveniences will result in the modelling of the contact interface between the punch and the half plane and the infinite property of the half plane and the singularity and the tip of the crack. Therefore, it is important and meaningful to present an analytical method to precisely solve this type of problem. Analytical solutions have some merits for the precision of the results and the physical meaning and appear to be one of the treasures.

The edge crack is assumed to be notably short and vertical. The crack occurs outside from the contact end. The maximum stress σ_x occurs along the surface of the half plane in this case. Therefore, this assumption of a vertical crack is reasonable. The positions of the crack initiation at the end of the punch with singularity were investigated for wedge-shaped punch [15].

The contact lengths, stress intensity factors of the crack and resultant moment on the contact region about the origin are investigated for different inclined angles for one or two ends in smooth contact. The knowledge of the inclined angle that makes the Mode I stress intensity factor of the short crack vanish is useful to analyze the initiating condition of the crack in the half plane. The stress intensity factors for the frictional coefficient are also investigated.

2. Presentation of the problem

Fig. 1(a) and (c) show an inclined circular punch acted by a vertical force and a horizontal force with one or two ends in smooth contact with a cracked half plane, respectively. The width of the punch is denoted by length a. In Fig. 1(a), position D of the right punch end of the contact region is fixed, and the origin of the xoy coordinate is located at the half-length $a/2$ of