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Technical Note

Singularity analysis for elastic clamped domains along an edge around which material properties depend on the angular coordinate

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ABSTRACT

The elastic isotropic solution of the displacements for three-dimensional clamped domains in the vicinity of an edge around which the material properties depend on the angular axis is represented by a family of eigen-functions (similar to 2-D domains) complemented by shadow-functions and their associated edge stress intensity functions (ESIFs). The explicit computation of the eigen-pairs and shadow functions for clamped isotropic domains where the elastic modulus, *E*, change smoothly in the material along the angular axis is presented herein. The computation method is semi-analytical and involves the p-finite element methods. Numerical examples are explicitly provided for cracks and V-notch edges and explore the eigenvalues as a function of the change in material properties in the angular direction. We demonstrate that the singular exponents may change considerably by changing the material properties variation in the angular direction and the eigen-functions are no longer neither symmetric nor asymmetric functions and therefore Mode *I* and Mode *II* may no longer be separated. These eigenpairs and their following shadow-functions are necessary to allow the extraction of the edge stress intensity functions.

1. Introduction

The Functional Graded Materials (FGM) are manufactured so that the material properties, although isotropic, change continuously in the material. These recent materials provide new design tools and are of increasing interest since the materials can be designed for specific function and applications. For example, the elastic modulus, *E*, may change smoothly in the material as a function of a coordinate.

The solutions to linear elastic problems in three-dimensional (3-D) polygonal domains in the vicinity of reentrant edges, when the material properties in the vicinity of the edges are constant or piecewise constant were studied in [6,12]. The solution is described in terms of special singular functions (eigen-functions and shadow-functions) depending on the geometry and the boundary conditions in the vicinity of the edge on one hand, and of unknown function of the coordinate along the edge (edge stress intensity functions) depending on the given body forces and tractions on the other hand.

In case of constant material properties, the eigen-pairs (eigen-values and eigen-functions) in the vicinity of crack tip or an edge may obtain in several techniques where a 2-D domain is considered. The techniques may be either analytical methods [10,4,3,1], semi-analytic methods [2] and numerical [5,9,13,11]. These methods are applicable to isotropic and anisotropic domains having constant material properties or piecewise constant (multi-material domains).

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The case of a non-constant material properties has been addressed in [7] where the eigen-functions and shadow-functions were computed in a semi-analytical approach in the vicinity of an edge. The eigen-functions and shadow-functions presented are part of the asymptotic solution in the vicinity of singular edges at which the material properties change continuously or piecewise continuously as a function of the angular axis θ. The singularity exponents in the asymptotic solution are sensitive to the material properties variation, and may obtain values considerably smaller than 1/2 (the crack exponent in homogeneous isotropic materials) depending on the angular variation of the Young modulus for example. Moreover, the tangential variation of the material properties result in a coupled mode situation, so that the mode I and mode II edges stress intensity functions become coupled although the loading conditions are pure mode I or pure mode II. The solution presented in [7] refer Traction Free boundary condition domains only.

The aim of this paper is to expand the asymptotic solution in the vicinity of singular edges at which the material properties change continuously or piecewise continuously as a function of the angular axis θ as presented in [7] to include cases where Homogeneous Dirichlet boundary conditions are considered. We derive the weak formulation for the computation of the eigenpairs (that construct the asymptotic expansion) and discretize this formulation so to allow the use the p-version of the FE method [8]. Numerical examples are then considered that demonstrate the dependence of the eigenvalues (singularity exponents) on the variation of the material properties for clamped domains.

The paper is organized as follows:

- We start with notations, defining the domain of interest and the linear elastic problem in the vicinity of an edge, and it's reduction to a 1-D problem for the computation of the eigenpairs in Section 2.
- A mathematical algorithm is presented for computing the eigen-pairs and the associated eigen-functions and shadows. The p-FE formulation is addressed, based on which numerical discretization of the continuous problem is applied in Section 3.
- Several numerical tests are presented in Section 4. The example problems demonstrate the accuracy and efficiency of the presented methods. Specifically we compute the eigen-pairs and shadow-functions associated with Homogeneous Dirichlet boundary conditions for a cracked and a V-notched domain with material properties which vary quadratically in θ. Both symmetric and anti-symmetric material properties in θ are considered.
- We conclude the paper with a summary and conclusions in Section 5.

2. The elastic solution for an isotropic problem in the vicinity of an edge

The asymptotic solution in the vicinity of an edge in an isotropic elastic domain having material properties depending on the angular angle when homogenous Dirichlet boundary conditions are considered is presented herein. The computation method for the elastic solution, presented as an asymptotic series of eigenpairs and shadows, is an extension of the method presented in [7] for traction free boundary conditions.

2.1. Notations and the differential equations

As a model, we choose a domain Ω such that only one straight edge $\mathcal{E}$ is present. The domain is generated as the product $\Omega = G \times I$ where I is the interval $[-1, 1]$, and G is a plane bounded sector of opening $\omega \in (0, 2\pi]$ and for simplicity assume it has a radius 1 (the case of a crack, $\omega = 2\pi$, is included), as shown in Fig. 1. Although any radius or interval I can be chosen, these simplified numbers have been chosen for simplicity of presentation.

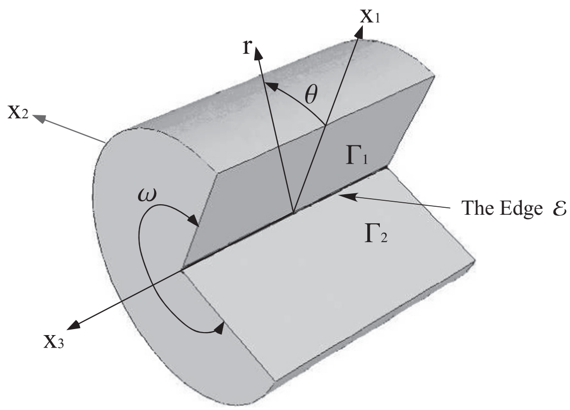

Fig. 1. Domain of interest Ω.