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Prediction of fracture initiation zones on the surface of three-dimensional structure using the surface curvature

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ABSTRACT

Using the well-known formula of Bažant for the prediction of structure nominal strength that depends on the structure characteristic size an expression for the local tensile strength is constructed and used in the maximal tensile stress criterion. This expression takes size effect into account not by using the characteristic size of a specimen but by using the minimal curvature radius of the specimen surface. The parameters of the expression for the local tensile strength depend on the material but do not depend on the geometry and the size of specimen. The distinctive feature of the *R*-criterion is its independence on the characteristic size of the specimen and therefore its convenience in practical application for 3D problems of complex geometry. It allows to take into account the difference of surface shape and strength among various points. The *R*-criterion is able to provide the accurate prediction of the static failure initiation of brittle three-dimensional complex components and to identify the location of initiated fractures. The tensile stress criterion enriched by the modified local tensile strength which depends on the minimal curvature radius in the analysed point is examined by the comparison of 2D and 3D numerical and experimental data.

1. Introduction

Material fracturing is a complex multi-stage process comprising elastic deformation of materials, microscopic defects in material, combining them into a microscopic incipient fracture and further fracture propagation. Understanding the nature of these processes is important both in fundamental science and in engineering practice. Mathematical modelling of material fracturing is one of the main methods of studying the mechanisms of such processes. The problem was analysed in numerous works, starting from the XIX century. The most comprehensive review is given in [1].

The most complete review of the latest outcomes of the research directed on the investigation of three-dimensional crack problems can be found in [2]. Linear elastic fracture mechanics (LEFM) is widely utilized for analysis of these problems. The application of the LEFM to the failure assessment of cracked specimens is justified in [3]. In this paper three characteristic zones around the tip of a crack are distinguished: the process zone, where the material is highly nonlinear; *K*-dominance region, where the linear elastic asymptotic stress field of the form $K/\sqrt{r}$ (the stress intensity factor divided by square root of the distance to the crack tip) is expected to be accurate; and the area of general stress state. In [4,5] it is proved that the conditions for the failure initiation are the functions of the magnitude of the stress intensity factor only and nothing else. A

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Nomenclature

d	size of the neighbourhood of body point [mm]
D	characteristic dimension of the structure or specimen [m]
D_b	parameter of Bažant's criterion and R-criterion [mm]
E	Young modulus [GPa]
F_s	function in fracture initiation criterion determined by stress-strain state of the body
F_m	function in fracture initiation criterion determined by material properties
$\mathbf{F}$	tensile force [N]
H	specimen thickness [m]
K_I	stress intensity factor mode I [$\mathrm{MPa}\sqrt{\mathrm{m}}$]
K_{Ic}	fracture toughness [$\mathrm{MPa}\sqrt{\mathrm{m}}$]
l_p	parameter of Bažant's criterion and R-criterion [mm]
n	parameter of Bažant's criterion and R-criterion
R	radius of surface curvature at the analysed point [m]
R_w	hole radius [m]
S	the surface of the specimen or the surface between the cavity and the elastic medium [m^2]
T	temperature [°C]
u_i	components of displacement vector [m]
$\mathbf{x}$	radius-vector of body point [m]
ε	parameter controlling the size of the fracture initiation zones
ε_{ij}	components of strain tensor
ν	Poisson ratio
σ_{ij}	components of stress tensor [MPa]
σ_t^∞	tensile strength [MPa]
σ_3	maximal stress at body point [MPa]
σ_N	nominal stress at peak load [MPa]
$\sigma_N(\mathbf{x})$	load required for fulfillment of fracture initiation criterion at the particular point $\mathbf{x}$ [MPa]
$\sigma_N^{\min}$	load required for fulfillment of fracture initiation criterion at least at one point of the body [MPa]
BEM	Boundary Element Method
FEM	Finite Element Method
MTS	Maximal Tensile Stress
SSS	Stress-Strain State
d-criterion	fracture initiation criterion based on averaged stress
K_I-criterion	fracture initiation criterion based on stress intensity factor of existing microfractures
R-criterion	fracture initiation criterion based on dependence of tensile strength on surface curvature radius

direct consequence of the selection of the stress intensity factor as a single fracture controlling parameter in LEFM is the scaling law of brittle fracture. LEFM predicts that the strength is proportional to the inverse square-root of the scale factor (the ratio of sizes of the two compared specimens). However, the LEFM predictions often turn out to be unsatisfactory. The thing is that the stress state near a crack tip is always three-dimensional, and an account for the 3D effects can shed light on fracture phenomena including the scaling law. The latest investigations of 3D effects associated with crack geometries and brittle fracture [6–9] just shed light on such important features, which cannot be recovered within the plane stress or plane strain theoretical framework of the theory of elasticity. A retrospective review of theoretical, numerical and experimental investigations of 3D effects at cracks has been provided by Kotousov et al. [8] and Pook [10,11].

In the majority of the papers devoted to 3D analysis of crack problems and fatigue crack growth modelling the finite element method (FEM) [12–16] is used. In these studies within the framework of the problem of the circular cylindrical plate with located at the centre of the cylinder the straight crack front it was found out that the region of 3D effects is confined to one-half of the plate thickness around the crack front before converging into a 2D plane stress field at a radial distance equal roughly to the plate thickness.

In [14] a simplified numerical-analytical technique is developed. This technique is capable to capture the general tendencies of the crack front shape evaluation and provide quite accurate qualitative and quantitative assessment of fatigue crack growth.

A comprehensive review devoted to coupling between the primary and local modes was published by Kotousov et al. [8]. The authors demonstrated significant and fundamental differences between the actual 3D stress state and the simplified classical 2D theories of quasi-brittle fracture leading to essentially different tendencies and predictions.

A special attention was given by He et al. [16] and Pook [10] to another characteristic feature of 3D solutions of elastic problems with cracks, called corner of vertex singularity. Despite on the limited volume of the cracked plate affected by the 3D corner singularity this 3D effect can have a significant influence on fracture and fatigue phenomena.