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Numerical estimation of strain intensity factors at the tip of a rigid line inclusion embedded in a finite matrix

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ABSTRACT

In this paper, the strain intensity factor at the tip of a rigid line inclusion, embedded in an isotropic matrix, is studied using analytical and numerical techniques. We first revisit the elasticity solution of a rigid inclusion embedded in an infinite elastic matrix using Stroh formulation as a basic framework. This study reveals that the strain intensity factor is appropriate for quantifying the magnitude of singularities at the inclusion tip. Next, we propose a numerical methodology, based on the reciprocal theorem, to calculate the strain intensity factor of the inclusion problem embedded in a matrix of finite geometry. The input to this method is the actual elasticity solution, which is obtained using finite element analysis (FEA). The FEA model is qualitatively validated by comparing FEA results with that of photoelasticity experiments. Finally, strain intensity factor in the case of finite geometry specimen is also estimated.

1. Introduction

In recent years, composite materials find wide applications in marine, aerospace and automotive sectors. Composite materials are combinations of two or more phases, e.g., fiber and matrix. They are used for structural applications in the form of both continuous and short fiber composite structures. The continuous fiber composite are replacing metallic structural parts, especially in aerospace industries, while the short fiber composites are used instead of plane polymeric material for electrical, packaging and automobile applications [1]. In short fiber composites, both fiber and matrix share the applied load, and the load transfer between the matrix and fiber happens via the fiber/matrix interface. As a consequence, the short fiber composites have superior strength and elastic stiffness over the parent polymeric material [2]. However, the fibers could also lead to singular stress field in the matrix near the tip of the fiber. If microvoids are present near the inclusion tip, the singular stress field will cause void growth, coalesce and micro-cracking. Moreover, the fiber-matrix interface is the weakest link in fiber reinforced composite laminates. Hence, it is important to understand the interaction between fiber and matrix in fiber composites from a damage mechanism perspective. As a first step towards understanding the mechanics of short fiber composites, the problem of a rigid line inclusion embedded in an elastic matrix is usually studied. The rigid line inclusion is assumed to play the role of a short fiber. This assumption is valid since (a) the thickness of the fiber is negligible in comparison to other dimensions of the composite and (b) the elastic modulus of the fiber is much larger than that of the matrix

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Nomenclature

$a,\ b$	major and minor radii of ellipse
$a_i,\ b_i$	material eigenvectors
$\boldsymbol{A},\ \boldsymbol{B}$	material eigenvector matrices
C_{ijkl}	components of elastic stiffness tensor
$\boldsymbol{d}^{\infty}$	applied far field strain vector
E	Young's modulus of the matrix material
z	anisotropic complex variable
$f(z)$	function of the anisotropic complex variable z
$\boldsymbol{I}$	unit matrix
K	bulk modulus of the matrix material
K_I	stress intensity factor
K_I^{ϵ}	strain intensity factor
$2l$	length of rigid line inclusion
n_j	unit vector normal to contour
p	material eigenvalue
$\boldsymbol{q}$	complex coefficient vector
$\boldsymbol{Q},\ \boldsymbol{R},\ \boldsymbol{T}$	matrices related to material properties
$r,\ \theta$	variables defining cylindrical coordinate reference system at an inclusion tip
$\boldsymbol{u}$	displacement vector
u_i	components of the displacement vector
$u_r,\ u_\theta$	displacement components in r and θ directions, respectively
$\boldsymbol{u}^{\infty}$	displacement vector at infinity
$\boldsymbol{t}^{\infty}$	traction vector at infinity
x_i	coordinates in the i^{th} direction, $i = 1,\ 2$
α	orientation of the rigid line inclusion
δ_{ij}	Kronecker delta
ν	Poisson's ratio
λ	order of singularity
μ	shear modulus of matrix
$\boldsymbol{\phi}$	stress function
$\boldsymbol{\phi}^{\infty}$	stress function vector at infinity
ϵ_{ij}	components of the strain tensor
σ_{ij}	components of the stress tensor
$\sigma^*_{ij},\ u^*_i$	components of the auxiliary stress tensor and displacement vector

Abbreviations

FEA	finite element analysis
SIF	stress intensity factor
CCD	charged coupled device

material. Analysis of the stress field and fracture parameters of a rigid line inclusion in an elastic matrix could provide interesting insights on the fiber-matrix interaction in short fiber composites.

Although the analytical solution for the rigid inclusion problem is known since [3], only a few studies exist on quantifying the associated fracture parameters. Atkinson [4] has provided displacement and stress field solution for ribbon like inclusion using complex variable approach. He has studied both rigid and elastic ribbon inclusion. Wang et al. [5] have derived the stress field at the tip of rigid line inclusion and attempted to quantify the elastic singularity using various stress intensity factors (SIF). The SIF for a rigid line inclusion, when the applied remote stress is transverse to the inclusion, was derived by Ballarini [6] using a singular integral and complex potential approach of Muskhelishvili [3]. Hasebe et al. [7,8] have considered a problem of kinked crack, which started from the tip of a rigid line inclusion. They also investigated the stress fields, SIF and resultant moment acting on the inclusion. Hurtado et al. [9] have solved for stress field of lamellar inhomogeneities such as crack, rigid line inclusion and elastic line inclusion using Eshelby's equivalent inclusion method. Li and Ting [10] studied the problem of a rigid line inclusion embedded in a general anisotropic elastic solid using Stroh formulation. Ni and Nemat-Nasser [11] showed that the solution for an inclusion could be derived directly from the solution of crack by using duality principle. Recently, the stress field and SIF for a line inclusion embedded in a matrix have been studied using photoelasticity by Noselli et al. [12]. The studies mentioned above focused only on analyzing the elastic fields and SIF for the rigid line inclusion in an infinite matrix. However, there are no investigations on the effect of finite geometry of the matrix on the SIF. Developing a numerical tool to quantify the singularity in the elastic fields near the rigid line inclusion tips is the primary focus of this work.

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