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Discontinuous Cell Method (DCM) for the simulation of cohesive fracture and fragmentation of continuous media

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A B S T R A C T

In this paper, the Discontinuous Cell Method (DCM) is formulated with the objective of simulating cohesive fracture propagation and fragmentation in homogeneous solids without issues relevant to excessive mesh deformation typical of available Finite Element formulations. DCM discretizes solids by using the Delaunay triangulation and its associated Voronoi tessellation giving rise to a system of discrete cells interacting through shared facets. For each Voronoi cell, the displacement field is approximated on the basis of rigid body kinematics, which is used to compute a strain vector at the centroid of the Voronoi facets. Such strain vector is demonstrated to be the projection of the strain tensor at that location. At the same point stress tractions are computed through vectorial constitutive equations derived on the basis of classical continuum tensorial theories. Results of analysis of a cantilever beam are used to perform convergence studies and comparison with classical finite element formulations in the elastic regime. Furthermore, cohesive fracture and fragmentation of homogeneous solids are studied under quasi-static and dynamic loading conditions. The mesh dependency problem, typically encountered upon adopting softening constitutive equations, is tackled through the crack band approach. This study demonstrates the capabilities of DCM by solving multiple benchmark problems relevant to cohesive crack propagation. The simulations show that DCM can handle effectively a wide range of problems from the simulation of a single propagating fracture to crack branching and fragmentation.

1. Introduction

A quantitative investigation of cohesive fracture propagation necessitates an accurate description of various fracture phenomena including: crack initiation; propagation along complex three-dimensional paths; interaction and coalescence of distributed multi-cracks into localized continuous cracks; and interaction of fractured/unfractured material. The classical Finite Element (FE) method, although it has been used with some success to address some of these aspects [1], is inherently incapable of modeling the displacement discontinuities associated with fracture. To address this issue, advanced computational technologies have been developed in the recent past. First, the embedded discontinuity methods (EDMs) were proposed to handle displacement discontinuity within finite elements. In these methods the crack is represented by a narrow band of high strain, which is embedded in the element and can be arbitrarily aligned. Many different EDM formulations can be found

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in the literature and a comprehensive comparative study of these formulations appears in Ref. [2]. The most common drawbacks of EDM formulations are stress locking (spurious stress transfer between the crack surfaces), inconsistency between the stress acting on the crack surface and the stress in the adjacent material bulk, and mesh sensitivity (crack path depending upon mesh alignment and refinement).

A method that does not experience stress locking and reduces mesh sensitivity is the extended finite element method (X-FEM) [3]. X-FEM, first introduced by Belytschko and Black [4], exploits the partition of unity property of FE shape functions. This property enables discontinuous terms to be incorporated locally in the displacement field without the need of topology changes in the initial uncracked mesh. Moës et al. [5] enhanced the initial work of Belytschko et al. [4] by including a discontinuous enrichment function to represent the displacement jump across the crack faces away from the crack tip. X-FEM has been successfully applied to a wide variety of problems. Dolbow et al. [6] applied XFEM to the simulation of growing discontinuity in Mindlin-Reissner plates by employing appropriate asymptotic crack-tip enrichment functions. Belytschko and coworkers [7] modeled evolution of arbitrary discontinuities in classical finite elements, in which discontinuity branching and intersection modeling are handled by the virtue of adding proper terms to the related finite element displacement shape functions. Furthermore, they studied crack initiation and propagation under dynamic loading condition and used a criterion based on the loss of hyperbolicity of the underlying continuum problem [8]. Zi et al. [9] extended X-FEM to the simulation of cohesive crack propagation. More recently, new orthotropic enrichment functions have been utilized to model interlaminar cracks in layered composites [10]. The main drawbacks of X-FEM are that the implementation into existing FE codes is not straightforward, the insertion of additional degrees of freedoms is required on-the-fly to describe the discontinuous enrichment, and complex quadrature routines are necessary to integrate discontinuous integrands.

Another approach widely used for the simulation of cohesive fracture is based on the adoption of cohesive zero-thickness finite elements located at the interface between the usual finite elements that discretize the body of interest [11–14]. This method, even if its implementation is relatively simple, tends to be computationally intensive because of the large number of nodes that are needed to allow fracturing at each element interface. Furthermore, in the elastic phase the zero-thickness finite elements require the definition of an artificial penalty stiffness to ensure inter-element compatibility. This stiffness usually deteriorates the accuracy and rate of convergence of the numerical solution and it may cause numerical instability. To avoid this problem, algorithms have been proposed in the literature [15] for the dynamic insertion of cohesive fractures into FE meshes. The dynamic insertion works reasonably well in high speed dynamic applications but is not adequate for quasi-static applications and leads to inaccurate stress calculations along the crack path.

An attractive alternative to the aforementioned approaches is the adoption of discrete models (particle and lattice models), which replace the continuum a priori by a system of rigid particles that interact by means of linear/nonlinear springs or by a grid of beam-type elements. These models were first developed to describe the behavior of particulate materials [16] and to solve elastic problems in the pre-computers era [17]. Later, they have been adapted to simulate fracture and failure of quasi-brittle materials in both two [18] and three dimensional problems [19–22]. In this class of models, it is worth mentioning the rigid-body-spring model developed by Bolander and collaborators, which dicretizes the material domain using Voronoi diagrams with random geometry, interconnected by zero-size springs, to simulate cohesive fracture in two and three dimensional problems [23–26]. Various other discrete models, in the form of either lattice or particle models, have been quite successful recently in simulating concrete materials [27–34].

Discrete models can realistically simulate fracture propagation and fragmentation without suffering from the aforementioned typical drawbacks of other computational technologies. The effectiveness and the robustness of the method are ensured by the fact that: (a) their kinematics naturally handle displacement discontinuities; (b) the crack opening at a certain point depends upon the displacements of only two nodes of the mesh; (c) the constitutive law for the fracturing behavior is vectorial; (d) remeshing of the material domain or inclusion of additional degrees of freedom during the fracture propagation process is not necessary. Despite these advantages the general adoption of these methods to simulate fracture propagation in continuous media has been quite limited because of various drawbacks in the uncracked phase, including: (1) the stiffness of the springs is defined through a heuristic (trial-and-error) characterization; (2) various elastic phenomena, e.g. Poisson's effect, cannot be reproduced exactly; (3) the convergence of the numerical scheme to the continuum solution cannot be proved; (4) amalgamation with classical tensorial constitutive laws is not possible; and (5) spurious numerical heterogeneity of the response (not related to the internal structure of the material) is inherently associated with these methods if simply used as discretization techniques for continuum problems.

The Discontinuous Cell Method (DCM) presented in this paper provides a framework unifying discrete models and continuum based methods. The Delaunay triangulation is employed to discretize the solid domain into triangular elements, the Voronoi tessellation is then used to build a set of discrete polyhedral cells whose kinematics is described through rigid body motion typical of discrete models. Tonti [35] presented a somewhat similar approach to discretize the material domain and to compute the finite element nodal forces using dual cell geometries. Furthermore, the DCM formulation is similar to that of the discontinuous Galerkin method which has primarily been applied in the past to the solution of fluid dynamics problems, but has also been extended to the study of elasticity [36]. Recently, discontinuous Galerkin approaches have also been used for the study of fracture mechanics [37] and cohesive fracture propagation [38]. The DCM formulation can be considered as a discontinuous Galerkin approach which utilizes piecewise constant shape functions. Another interesting feature of DCM is that the formulation includes rotational degrees of freedom. Researchers have attempted to introduce rotational degrees of freedom to classical finite elements by considering special form of displacement functions along each element edge to improve their performance in bending problems [39,40]. This strategy leads often to zero energy deformation modes and