ELSEVIER

Comput. Methods Appl. Mech. Engrg. 190 (2001) 2017–2030

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

A new algorithm for solving some mechanical problems

G.W. Wei *

Department of Computational Science, National University of Singapore, Singapore 117543, Singapore

Abstract

This paper explores the utility of a discrete singular convolution algorithm for solving certain mechanical problems. Benchmark mechanical systems, including plate vibrations and incompressible flows, are employed to illustrate the robustness and to test accuracy of the present algorithm. Numerical results indicate that the present approach is very accurate, efficient and reliable for solving the aforementioned problems.

1. Introduction

Since many practical problems in science and engineering are either extremely difficult or impossible to solve by conventional analytical methods, numerical simulations play a more and more important role in handling these problems. The advent of high-performance computers has given tremendous impetus to all numerical methods for solving science and engineering problems. Although there has been a great deal of achievement in developing accurate, efficient and robust computational methods, finding numerical solutions for partial differential equations (PDEs) is still a challenge owing to the presence of possible singularities and/or homoclinic manifolds that induce sharp transitions in the solutions. The presence of these phenomena can be extremely sensitive to numerical algorithms and can easily lead to numerically induced spatial and/or temporal chaos [1]. The conventional approaches to these problems may be classified as either global methods [2–6] or local methods [7–16]. Global methods are highly localized in their spectral representations, but are unlocalized in the coordinate representation. By contrast, local methods have high spatial localization, but are delocalized in their spectral representations. Moreover, the use of global methods is usually restricted to structured grids, whereas, local methods can be implemented to block-structured grids and even unstructured grids. In general, global methods are much more accurate than local methods, while the major advantages of local methods are their flexibility in handling complex geometries and boundary conditions. In ordinary applications, it is relatively safe and efficient to use either a global method or a local one for numerically solving an ordinary differential equation or a partial differential equation. However, when a differential equation has singularities and/or homoclinic orbits, neither the global methods nor the local methods can be applied without numerical instabilities. The global methods lose their accuracy near the singularities due to local high frequency components. The local methods have to be implemented in an adaptive manner, which greatly limits their accuracy and requires extremely small (spatial and/or temporal) mesh sizes. In many situations, the rate of convergence of a numerical method simply cannot match the divergent rate of the problem under study near a singularity. It is desirable to have a method that has both spectral and spatial localization, and is thus locally smooth and asymptotically

* Tel.: +65-874-6589; fax: +65-774-6756.
E-mail address: cscweigw@nus.edu.sg (G.W. Wei).

0045-7825/01/$ - see front matter
PII: S0045-7825(00)00219-X

decaying in both spectral and coordinate spaces. Particularly, such a method has the feature that combines global methods' accuracy with local methods' flexibility.

The discrete singular convolution (DSC) algorithm [17] was proposed as a potential approach for numerical realization of Hilbert transform, Abel transform, Radon transform, and delta transform. These transforms are essential to many practical applications, such as computational electromagnetics, computed tomography, molecular potential surface generation and dynamic simulation. The DSC algorithm has been tested for its applications to stochastic process analysis [17], nanoscale pattern formation of complex systems [18], homoclinic orbit of the Sine–Gordon singularity [19], and quantum eigenvalue problem of the Schrödinger equation [20]. The underlying mathematical structure for the DSC algorithm is the theory of distributions [21].

The purpose of this paper is to explore the utility and test the reliability of the DSC algorithm for mechanical applications. To this end, we consider two types of problems, plate vibrations and incompressible flows. This paper is organized as follows: The DSC algorithm is reviewed in Section 2. Some relevant parts of the algorithm are described in a greater detail than the original paper. Vibration analysis by the DSC algorithm is presented in Section 3. Eigenfunctions and eigenvalues of a rectangular plate and a circular plate are studied. Section 4 is devoted to fluid flow applications. We consider two test examples, the Taylor problem and a double shear layer flow, to illustrate the accuracy and robustness of the DSC approach for flow simulations. This paper ends with a conclusion.

2. Theory and algorithm

Singular convolutions are essential to many science and engineering problems, such as electromagnetics, Hilbert transform, Abel and Radon transforms. DSC is a general approach for the numerical realization of singular convolutions. By appropriate construction or approximation of a singular kernel, the discrete singular convolution can be an extremely efficient, accurate and reliable algorithm for practical applications [17].

It is most convenient to discuss singular convolution in the context of distributions. We denote T a distribution and $\eta(t)$ an element of the space of test functions. A singular convolution can be expressed as

$$F(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)\,\mathrm{d}x. \tag{1}$$

Here $T(t-x)$ is a singular kernel. Depending on the form of the kernel T, the singular convolution is the central issue for many science and engineering problems. For example, singular kernels of the Hilbert type have a general form of

$$T(x) = \frac{1}{x^n}, \quad (n > 0). \tag{2}$$

Here, kernels $T(x) = 1/x^a, (0 < a < 1)$ define the Abel transform which is closely connected with a generalization of the tautochrone problem. Kernel $T(x) = 1/x$ commonly occurs in theory of linear response, signal processing, theory of analytic functions, and the Hilbert transform. Its three-dimensional version is important to the theory of electromagnetics. $T(x) = 1/x^2$ is the kernel used in tomography. Other interesting examples are singular kernels of the delta type

$$T(x) = \delta^{(n)}(x), \quad (n = 0, 1, 2, \ldots). \tag{3}$$

Here, kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x),\ (n = 1, 2, \ldots)$ are essential for numerically solving differential equations. However, a common feature of these kernels is that they are singular, i.e., they cannot be directly digitized in computers. In this regard, the singular convolution, (1), is of little numerical merit. To avoid the difficulty of using singular expressions directly in computers, sequences of approximations (T_α) of the distribution T can be constructed