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Geometrically non-linear multi-layer beam with interconnection allowing for mixed-mode delamination

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ABSTRACT

In this work we assess the extent to which a beam model is suitable for the finite-element analysis of composite structures undergoing a large-displacement delamination process. We lay down the necessary theory needed for the geometrically non-linear analysis using Reissner's beam theory for the layers to be applied to layered structures involving dual-mode damage-type bi-linear constitutive law for the interconnections, run a number of representative examples and compare the results to those obtained using a geometrically linear analysis. The formulation is given in a general form where the number of layers and nodes of the beam finite elements is arbitrary. To solve numerical problems, the equilibrium of which is necessarily more complex and demanding to satisfy than in the geometrically linear case, the standard cylindrical arc-length procedure is used only when there is no damage at the interconnection. When damage at the interconnection occurs, the standard arc-length method has been modified so that in each load step the converged solution is required to result in an increase in the total damage of the system. It is concluded that the geometrically linear formulations can be used with satisfactory accuracy only in limited number of cases where displacements and rotations remain small.

1. Introduction

Structures composed of multiple layers can be found in many areas of engineering as well as in nature. The most prevalent failure mechanism of such structures is delamination in which the connection between the layers is being progressively damaged due to cracking and is eventually completely lost. Obviously, this failure mechanism is very complex for a variety of reasons.

To start with, it exhibits overall structural softening upon reaching a particular strength of the interconnection [1] and in order to assess this strength it becomes necessary to invoke the fundamental energy principles from the theory of fracture mechanics [2]. The actual softening may be described exponentially, as in the linear fracture mechanics (see e.g. [3]) or as a linear or multi-linear curve, often used in numerical analyses. The global manifestation of post-critical softening may often become apparent in considerably larger overall displacements compared to those in the pre-critical range necessitating a geometrically non-linear structural analysis.

In addition, instead of considering the delamination stress at the crack tip as infinite, which follows from the principles of linear fracture mechanics [4], in real practical problems it becomes necessary to recognise that the fracturing process is governed by a finite stress distribution over a small region around the crack tip, the so-called "process zone" in Barenblatt's

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cohesive zone models (CZM) [5]. The cohesive zone models enable the stresses to 'straddle' a narrow crack and describe a variety of physical phenomena rather well, from generation and localisation of a principal crack [6–8] to aggregate interlocking in concrete structures [9].

Also, a crack between two layers may occur for different reasons leading to the so-called Mode I, II or III openings (normal to the crack surface, or tangential to it due to slippage or tearing) [1]. Obviously, these may not be considered separately since even a limited damage in a particular mode always comes as a consequence of some underlying physical re-arrangement of particle bonds on a sufficiently small scale which necessarily reduces also the strengths in the other modes. It thus becomes necessary to define a certain scalar measure of overall damage (see e.g. [10]), which involves contribution from all possible modes and governs the phenomenon of damage-induced strength reduction in all the modes.

When modelling engineering problems we are naturally led by the demands of (i) accuracy and (ii) computational efficience, which need to be met to within a prescribed measure and in some sense optimised. For the class of problems analysed here, in our previous work [11] it has been shown that using beam finite elements instead of 2D solids for planar geometrically linear delamination gives results of comparable accuracy using significantly less degrees of freedom. Such elements do not appear to be as wide-spread in this type of analysis as the solids, and it is thus argued that they should be considered as a valid alternative in a variety of situations, including mixed-mode delamination. The efficiency of multi-layer beam finite elements in comparison with commonly used 2D solids has been shown also in authors' previous work [12] where the connection between the layers was assumed to be absolutely rigid (see also [13,14]).

In this work we attempt to assess the extent to which the beam model and, more generally the geometrically linear set-up itself, are applicable to the analysis of the composite structures undergoing a delamination process. Not unexpectedly, such structures are usually designed to take advantage of the particular properties of the materials forming the composite without being damaged in the operational state. However, if we want to trace the post-critical equilibrium path after the process of delamination has initiated, possibly all the way up to full rupture, we have to recognise that the ratio between the displacement and the loading magnitudes may increase considerably. There also exist such delamination phenomena, e.g. peeling, in which the displacements are of the order of magnitude of the geometry of the problem analysed.

In such situations, obviously, geometrically linear analysis may not return the results representative of the real behaviour of the problem analysed. Given the complexity of the delamination process, it is not always possible to tell in advance if the geometrically non-linear effects may not in fact become considerable even for deformation magnitudes which we may be tempted to intuitively classify as 'small'.

In this work we will lay down the necessary theory needed for the geometrically non-linear analysis using Reissner's beam theory for the layers to be applied to layered structures involving dual-mode damage-type bi-linear constitutive law for the interconnections. In order to assess the need for the geometrically non-linear analysis presented, we will run a number of representative examples and compare the results to those obtained using a geometrically linear analysis.

2. Problem description

Geometry of deformation of a multi-layer beam is described in [11] and here we reproduce it for reference. An initially straight multi-layer beam composed of n layers and $n-1$ interconnections is considered. An arbitrary interconnection α is placed between layers i and $i+1$.

Material co-ordinate system of each layer is defined by an orthonormal triad of vectors $\boldsymbol{E}_{1,i}, \boldsymbol{E}_{2,i}, \boldsymbol{E}_{3,i}$, with axes $X_{1,i}, X_{2,i}, X_{3,i}$ (see Fig. 1). The axes $X_{1,i}$ are parallel with the layer's edges and mutually ($\boldsymbol{E}_1 = \boldsymbol{E}_{1,i}$ and $X_1 = X_{1,i}$) coincide with the reference axes of each layer. The position of a reference axis over the layer's height $a_i \in \langle 0, h_i \rangle$ may be chosen arbitrarily, where h_i is the layer's height. However, in [11] it was shown that the position of the reference axis may influence the numerical results. The cross-sections of all layers have a common vertical principal axis X_2 defined by a base vector $\boldsymbol{E}_2 = \boldsymbol{E}_{2,i}$ (a condition for a planar deformation). Note that, according to Fig. 1, the co-ordinate $X_{2,i}$ is different for each layer i. Axes $X_{3,i}$ are mutually parallel ($X_3 = X_{3,i}$ and $\boldsymbol{E}_3 = \boldsymbol{E}_{3,i}$), but they do not necessarily coincide with the horizontal principal axes of the layers' cross-sections. The first and the second moment of area of the layer's cross-section with respect to axis $X_{3,i}$ are defined as

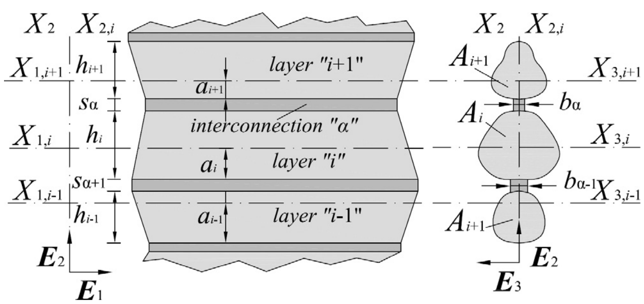

Fig. 1. Position of a segment of a multi-layer beam with interconnection in the material co-ordinate system.