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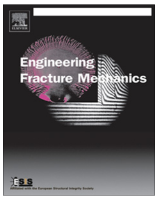

A J-integral approach in characterizing the mechanics of a horizontal crack embedded in a cantilever beam under an end transverse force

Xiaomin Fang, Panos G. Charalambides *

Department of Mechanical Engineering, University of Maryland, Baltimore County, Baltimore, MD 21250, USA

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ABSTRACT

For a cantilever beam with an embedded sharp crack and subjected to an end transverse force, the J-integral approach was employed in developing analytical estimates of the energy release rate made available to the left and right crack tip. Finite element studies also revealed that mode II conditions dominate the tip regions of such a crack (Fang and Charalambides, 2015). Thus, analytical estimates of the mode II stress intensity factor dominating each of the crack tip regions are also obtained. The analytical energy release rate predictions are compared to 2-D finite elements for a broad range of crack depths and crack center location along the beam axis. Using energy considerations, rotary spring stiffness estimates employed in Charalambides and Fang (2016a,b) in the development of a four-beam model obtained. The outcomes of the methodology used in this work provide strong encouragement in extending the method to heterogeneous composite laminates containing delamination cracks and subjected to a combination of applied loadings. Given the predominately mode II nature of the crack considered, the solutions developed herein may be used in characterizing mode II interface fracture for bonded homogeneous layered beams.

1. Introduction

A crack embedded in an elastic, homogeneous and isotropic system is known to propagate in a direction of maximum energy release rate experienced by incipient kink cracks (i.e., cracks of "zero length" compared to the cracked length) at the crack tip [4–7]. For example, under mode I crack surface opening conditions, such an incipient kink crack directly ahead of the crack tip, i.e., residing on the crack plane, experiencing maximum energy release rate and such it is the one activated during crack growth initiation thus confining the macro-crack path to the crack plane as shown schematically in Fig. 1a. However, under pure relative crack surface sliding conditions, i.e., pure mode II loading conditions (see Fig. 1b), local maximum in the energy release rate is experienced by a kink crack at approximately $\theta^k \approx 70.3^\circ$ clockwise from the crack plane for a positive K_{II}, where K_{II} is the mode II stress intensity factor (SIF). In the latter case, a crack embedded in a homogeneous and isotropic medium would kink out-of-plane along the maximum energy release rate kink direction as shown in Fig. 1b. Interestingly, the kink crack path direction predicted using a maximum energy release rate criterion coincides with a path that is perpendicular to the maximum principal stress near the crack tip. Regardless of the criterion used, embedded cracks in homogeneous and isotropic systems would not propagate in their original plane if a mode II component exists. Given the

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* Corresponding author.
E-mail address: panos@umbc.edu (P.G. Charalambides).

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Nomenclature

x	coordinate along the beam length
x_C	coordinate of the sharp crack center
y	coordinate along the beam height, or analytical deflection along neutral axis of the slender beam in $x-y$ plane
y_C	coordinate of the sharp crack center
a	half of crack length
l	crack length
L	beam length
l_1	length of sub beam, between the left tip of the horizontal crack to the beam fixed end
l_3	length of sub beam
h	beam height
t	beam width
F	concentrated force
P	load
E	elastic modulus
ν	Poisson's ratio
G	shear modulus
N	axial force
V	shear force
M	bending moment
I	cross-sectional second moment of inertia
A	cross-sectional area
Γ	contour
J	J-integral
C	beam compliance
$\Im$	energy release rate
$\Im_c$	characteristic energy release rate
t	subscript or superscript denoting the quantity for the top beam
b	subscript or superscript denoting the quantity for the bottom beam
A	subscript denoting the quantity at Section A
B	subscript denoting the quantity at Section B
C	subscript denoting the quantity at the crack center or at the section passing through the crack center
^{MM}	superscript denoting the quantity derived from analytical four beam model
$COMP$	superscript denoting the quantity derived from compliance method
cr	superscript denoting the crack related quantity
λ	transition region length parameter, dimensionless
σ	stress
ε	strain
δ	deflection
Δ	difference
φ	rotation of beam cross section
ϑ	rotation of beam cross section with shear effects
k	timoshenko shear constant
θ	crack orientation
K_I	mode I stress intensity factor (SIF)
K_{II}	mode II stress intensity factor (SIF)
h_t	distance between the horizontal crack plane and the beam top surface, also known as crack depth
h_b	distance between the horizontal crack plane and the beam bottom surface
Φ	strain energy
w	strain energy density

findings reported elsewhere [1], one would have difficulties to envision the conditions under which mode II crack such as a horizontal crack embedded in a cantilever beam under an end force loading, would initiate and grow in plane as modeled in this study over the lifetime of a component. In all likelihood, cracks developed during the life of a component such as a cantilever beam would most likely initiate and grow in the tensile region normal to the bending stress. Thus, one would expect to see such cracks to be oriented perpendicular to the beam axis, most likely originating from a surface flaw and growing vertically towards the beam neutral axis thus forming a typical edge mode I crack. It is thus not surprising that many crack detection studies [8–12] have indeed been conducted for edge cracks under mode I conditions for which empirical solutions for the stress intensity factor do exist. However, no such rigorous fracture mechanics methods have been used in crack detection studies involving a horizontal, fully embedded crack, perhaps due to their focus on detecting damage in homogeneous