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Investigation on Filippi's stress equation for V-shaped notch with rounded vertex Part I: Mode I stresses

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A B S T R A C T

In the present paper, in order to investigate systematically the effectiveness of Filippi's stress equation applied at evaluating stresses generated in a rounded V-shaped notch with a finite depth, the stresses generated in a semi-infinite plate with rounded V-shaped notch subjected to a uniform tension load at remote are used as a base data to examine the applicability of Filippi's stress equation. It is found that Filippi's equation gives an excellent approximation to the stresses along the notch bisector for rounded V-shaped notches. The generalized stress intensity factor $K^V_{\rho,I}(r)$ is almost constant along the notch bisector in the region where the stresses are governed by the singular stress field in a sharp notch; the peak discrepancy between Filippi's equation and the numerical analysis is less than 7% when the notch opening angle $\gamma = 0^\circ, 2.5\%$ when $\gamma = 30^\circ$ and 5% when $\gamma = 90^\circ$ and 120°. Also it is found that Filippi's equation is effective in evaluating the stresses along direction different from the notch bisector, however, as the calculation angle θ with respect to the notch bisector increases, the discrepancy between Filippi's equation and the numerical analysis becomes larger. Therefore, it is advisable to limit calculation points of stress within a range of $\theta \leqslant 30^\circ$. Moreover, values of the maximum stress σ_{max} predicted by various techniques are also examined using the numerical-analysis results of the semi-infinite plate with a V-shaped notch, and an analytical approach for predicting the constant a_1 in Filippi's equation is proposed.

1. Introduction

V-shaped notches as structure components usually have its rounded vertex which can be seen as a part of circle arc with curvature radius ρ not being zero. In order to evaluate destruction of such a rounded V-shaped notch, it is important to quantitatively analyze the stress distribution capable of including the influence of notch root radius. For this reason, studies on the stress distribution near the rounded notch tip were carried out by many researchers until now [1–13].

Filippi et al. [10] has proposed a stress approximate expression for a rounded V-shaped notch subjected to the mode I and the mode II loadings. Although the stresses near the notch tip can also be analyzed numerically by computational technology, such as FEM, the approximate stress expression proposed by Filippi et al. [10], called Filippi's stress equation here, provides a rapid evaluation of stresses and is very useful in engineering practice for studying influence of each geometric

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Nomenclature

a_1	constant in Filippi's stress equation
K_I^V	mode I notch stress intensity factor for a sharp V-notch
$K_{\rho,I}^V$	mode I notch stress intensity factor for a blunt V-notch
r	distance measured from the coordinate origin
r_0	distance between the coordinate origin and the notch tip
t	notch depth
γ	notch opening angle
θ	direction angle in the polar coordinate system
λ_1	eigenvalue in mode I (singularity exponent)
μ_1	eigenvalue in mode I (real parameter)
ρ	notch radius
σ_r	radial stress
$\sigma_{r\theta}$ or $\sigma_{r\theta}$	in-plane shear stress
σ_θ	tangential stress
σ_{max}	max maximum tangential stress at notch root
σ^∞	uniform tension stress at remote

parameter systematically. Therefore, since Filippi's stress equation was proposed, it has been applied in many studies and its validity has been shown [11,14–17].

Desire for obtaining a more exact understanding of Filippi's stress equation and its application at evaluation of fracture of notches stimulates the author to add a further investigation on Filippi's stress equation.

The Filippi's stress equation is theoretically a solution to a V-shaped notch where the notch depth can be regarded as infinite, that is, the ratio ρ/t of the notch root radius ρ to the notch depth t is infinitely small. On the other hand, the depth of an actual notch used in component is usually of a finite value. So, even the notch root radius may be very small, the ratio ρ/t of the radius to depth would take a finite quantity. Therefore, in order to make the Filippi's stress equation apply to the destructive problem of notches, it is necessary to investigate the effectiveness of Filippi's stress equation applied at evaluating the stresses generated in a rounded V-shaped notch with a finite depth. Although such studied was already conducted, systematic investigation is insufficient. So, in this study, based on analysis results of highly precise body force method (BFM), stresses generated in a semi-infinite plate with rounded V-shaped notch subjected to a uniform tension load at remote, as shown in Fig. 1, are used as a base data to investigate the applicability of Filippi's stress equation. In Fig. 1, the notch depth and opening angle are denoted by t and γ, respectively.

In Filippi's stress equation a constant a_1 is included, which must be determined for finally giving stress distribution near the notch tip. The constant a_1 is directly linked to the maximum stress σ_{max} at the notch bottom. There exist many tech-

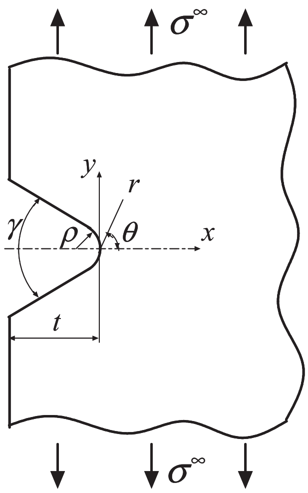

Fig. 1. Rounded V-notch in a semi-infinite plate subjected to a remote load σ^∞.