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Letter to the Editor

Discussion on the interpretation of scratch tests with size effect law

1. Introduction

Scratch tests provide a convenient and reliable way to estimate the strength properties of rocks [1]; however, the tests' applications in assessing fracture properties remain subject to scientific debate [2–7]. Akono et al. proposed a scratch test model to determine the fracture toughness of quasi-brittle materials within the framework of linear elastic fracture mechanics (LEFM) [2]. Lin and Zhou argued that the failure mode of typical tests with shallow cutting depths (e.g., less than 1 mm) is not governed by LEFM, by introducing Bažant's size effect law (SEL) to interpret the ductile-brittle failure mode transition in scratch tests [5,6,8]. Akono et al. extended the application of the SEL in scratch tests to assess fracture properties in both 2D to 3D conditions by considering different cutting widths, arguing that the fracture toughness obtained with the SEL is very close to that obtained based on LEFM [3,4]. Le and Detournay recently pointed out that Akono et al.'s interpretation of scratch tests based on LEFM violates several fundamental issues of LEFM [7]. The objective of this Letter is to show that Akono et al.'s interpretation of scratch tests based on the SEL violates the fundamental strength theory and thus their evaluation of fracture toughness is questionable.

Bažant's SEL bridges the gap between strength theory and LEFM for quasi-brittle materials. It can be expressed as [8]:

$$\sigma_N = \frac{B\sigma_y}{\sqrt{1 + d/d_0}} \tag{1}$$

where σ_N is the nominal strength defined as $\sigma_N = c_N F/(wd)$, c_N is a coefficient introduced for convenience, F is the peak force, w is the structure width, d is the structure size, σ_y is a yield strength, B is a coefficient, and d_0 is the critical structure size.

The limit value of B, denoted as B^*, can be interpreted as the dimensionless nominal strength for very small structure sizes:

$$B^* = \frac{\sigma_N|_{d \to 0}}{\sigma_y} \tag{2}$$

B^* can be conveniently obtained based on strength theory for tests on notched structures under simple loading conditions. For example, $B^* = 1 - \alpha$ for direct tension tests when $c_N = 1$, where α is a dimensionless crack length defined as the ratio of the initial crack length to structure size. $B^* = 3(1 - \alpha)^2$ for three point bending tests when c_N is one and half times the span-to-depth ratio [9].

Even though the loading conditions of scratch tests are very different from those of direct tension and three point bending tests, experimental and numerical data have been found to align closely with the SEL by treating the depth of cut as a measure of structure size [5,10,11]. Two parameters, B and d_0, are obtained through regression analysis by interpreting scratch tests with the SEL. In order to justify the regression results, the regressed value of B is compared with its limit value, which is obtained by considering a 2D plane strain problem as shown in Fig. 1. This idealized problem is described by three sets of parameters. One set characterizes the geometry: the rake angle θ and depth of cut d; a second set describes a cohesive-frictional material (e.g., rock) that follows the Mohr-Coulomb failure criterion: the internal friction angle φ and cohesion c (or equivalently, the unconfined compressive strength $\sigma_c = 2c\cos\varphi/(1 - \sin\varphi)$); a third set defines the interface: the interface friction angle ψ and total force F. The limit value of the dimensionless nominal strength is defined as:

$$\Sigma^* = \frac{F_T^*}{\sigma_c w d} \tag{3}$$

where F_T^* is the limit value of cutting force, w is the width of cut and it is unity in the 2D condition.

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Nomenclature

B, B^*	coefficient in size effect law and its limit value
c	cohesion
c_N	coefficient
d	structure size or depth of cut
d_0	critical structure size or critical depth of cut
F, F_T, F_V	total force, cutting force, vertical force
F_T^*	limit value of cutting force
$\overline{F}_T^p$	mean peak cutting force
i_c	force inclination factor
N_c	bearing capacity coefficient
s, s_a, s_p	stress parameters
w	structure width or width of cut
α	dimensionless crack length
ψ	interface friction angle
φ	internal friction angle
σ_c, σ_y	unconfined compressive strength, yield strength
σ_n	normal contact stress
σ_N	nominal strength
$\sigma, \sigma_x, \sigma_z$	normal stresses
τ	shear stress
θ	rake angle
Π^*	limit value of dimensionless contact stress
Σ, Σ^*	dimensionless nominal strength and its limit value
Ψ_a, Ψ_p	inclination angles of major principal stresses
$\Delta\Psi$	fan angle

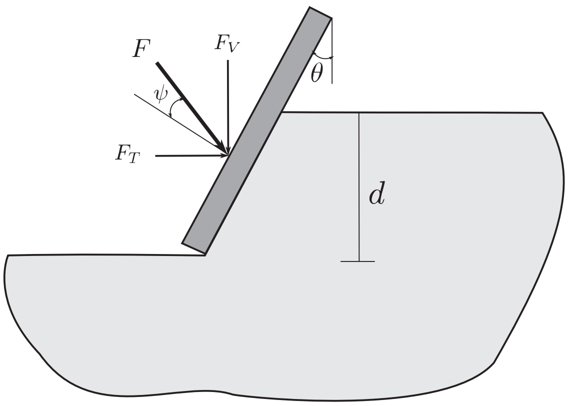

Fig. 1. Idealized representation of a plane strain scratch test.

The slip-line method that has been used to calculate the limit load of cohesive-frictional materials is adopted here to determine the lower bound solution of the dimensionless nominal strength [12,13]. The slip-line field in the active, fan, and passive zones is illustrated in Fig. 2. Based on the detailed derivations found in Appendix A [12–15], the limit value of the dimensionless nominal strength is:

$$\Sigma^* = \frac{\cos(\psi+\theta)}{\cos\theta\cos\psi}\frac{1-\sin\varphi}{2\cos\varphi} i_c N_c \tag{4}$$

where i_c is a force inclination factor between 0 and 1 with its expression given in Appendix A [13], and N_c is a bearing capacity coefficient [12].

The lower bound solution of the dimensionless nominal strength is identical to Merchant's upper bound solution, i.e., $\Sigma^* = 1$, for a special case with $\theta = 0$ and $\psi = 0$ [16]. Fig. 3 shows that Σ^* is of order $O(1)$ and it is around 2–3 for typical tests with $\varphi = 20$–40°, $\psi = 20^\circ$, and $\theta = 15^\circ$. The interface friction angle ψ of 20° is used here since extensive tests conducted on