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Size effects on double cantilever beam fracture mechanics specimen based on strain gradient theory

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ABSTRACT

This paper investigates large deformation of a cantilever beam which is further employed to study the fracture behavior of double cantilever beam (DCB), based on strain gradient elasticity theory. Root effect of the DCB is also included for modelling and analyses. The numerical solutions of maximum tip deflection and strain energy release rate are presented. Results demonstrate that the consideration of large deformation is crucial at small scale, especially for more slender beams, as the bending behavior of the beam in that case is different from the classical results. The strain gradient and root effects of the DCB are more prominent when thickness of the beam is less than the material length scale parameter. The strain gradient model demonstrates significant stiffening behavior at the smaller scale. In general, the root effect may not be neglected if the length to thickness ratio of the beam is smaller. Overall, the strain energy release rate of the gradient model, even with the incorporation of root part, remains less than that of non-gradient model. This conclusion is entirely different from the classical method that neglects the uncracked part of the DCB.

1. Introduction

Mechanical structures, such as beams are often subjected to large deformation which tends to induce geometrical non linearity, such that the relation between applied force and the curvature becomes non-linear. This non-linear behavior will effectively change the stiffness of the structure. This response is shown to be dominant in literature for the case of clamped-clamped and simply supported beams. In contrast, the non-linear response of cantilever beam has received less attention comparatively [42]. Cantilever beams used in micro and nanoelectromechanical (MEMS & NEMS) switches often undergo geometrical non-linearity. Using linear theory, the error in strain energy release rate is found to be larger than 30%, as shown by mixed mode bending (MMB) tests. However, with the consideration of geometric nonlinearity, the redesigned MMB apparatus demonstrate the error to be less than 3% [36,46]. The conventional mathematical treatment of analysing a cantilever beam, that assumes small deformation does not hold many complexities and hence exact solution can be derived quite comfortably. Nevertheless, with the addition of large deformation (geometrical non linearity), the problem involves the non-linear term that are difficult to solve analytically. In the past, several efforts have been devoted to address this issue, for instance, the analysis of large deformation of cantilever beams may be found in work of Beléndez et al. [6] and Landau and Lifshitz [29]. It was shown that the results, with the consideration of large deformation, were in better agreement with

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Nomenclature

A	cross-sectional area of the beam
a	length of the beam
b	width of the beam
DCB	double cantilever beam
E	Young's modulus
e_x	axial strain
F	applied force
G^s	shear modulus
G	strain energy release rate (large deformation)
G_o	classical strain energy release rate
G_g	strain energy release rate of a strain gradient model
h	height of the beam
I	moment of inertia of the beam
l	material length constant related to volumetric elastic strain energy
l'	material length constant related to surface elastic strain energy
M	bending moment of a beam
N	resultant force along the x-direction
$MEMS$	microelectromechanical systems
MMB	mixed mode bending
$NEMS$	nanoelectromechanical
p	normal stress on the $z = 0$ plane
Q	shear force on the beam cross-section
q	shear stress on the $z = 0$ plane
R	ratio of the strain energy release rate contributed by the uncracked part the cracked part of the DCB
s	arc length along the deformed beam
U	potential energy density
U_1	total strain energy
X	horizontal deflection
Y	vertical tip deflection (large deformation)
Y_o	classical tip vertical deflection
Y_g	vertical tip deflection of strain gradient model

Special symbols

u_o	displacement of the beam along the x direction
v	Poisson ratio
φ	angle of rotation of a beam
τ_x	Cauchy stresses
μ_x	double stresses
σ_x	total stresses

the experimental data upon comparison with the classical theory. Meanwhile different numerical techniques are also used to obtain large-deformation solutions for cantilever beam [35].

The bending behavior of cantilever beam in literature is often employed to study the fracture behavior of double cantilever beam (DCB) [16,39]. The DCB is typically considered to be consist of two cantilever beams attached with the root part (uncracked part) and is used broadly in experiments to determine the Mode I fracture toughness of the materials. In the tests of DCB, Devitt et al. [11] found that the effect of geometric nonlinearity on the mode I fracture toughness of composite materials is suffice for long cracks; similar findings are also mentioned in other reference [47]. Furthermore DCB is the most widely used test configuration for the study of crack propagation and arrest for composite materials and adhesives. Either in theoretical studies or experimental investigations, the DCB specimen has been found to be quite convenient to determine the mode I fracture toughness of homogenous, composite laminates and adhesively bonded materials. Sebaey et al. [37] used numerical methods to investigate the asymmetric crack growth in double cantilever beam tests of multidirectional composite laminates. The solution was the extension of the work previously conducted by Kanninen [24]. De Moura et al. [10] employed numerical and experimental methods to investigate the fracture characteristics of double cantilever wood beam specimen. De Morais [9] developed a new analytical method to compute mode I critical strain energy release rates unaffected by fibre bridging. Wang and Wang [45] derived the closed-form solutions of the strain energy release rate and stress inten-