Engineering Fracture Mechanics 165 (2016) 87–97

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Geometric description of fracture surface features in isotropic brittle solids

Roberto Dugnani [a,*], Ricardo J. Zednik [b]

[a] University of Michigan, Shanghai Jiao Tong University Joint Institute, UM-SJTU Joint Institute, 800 Dongchuan Road, Shanghai 200240, China
[b] École de Technologie Supérieure, Université du Québec, Montréal, Canada

ARTICLE INFO

Article history:
Received 30 October 2015
Received in revised form 9 August 2016
Accepted 10 August 2016
Available online 31 August 2016

Keywords:
Fracture mechanism
Glass material
Fractography

ABSTRACT

Fracture surface contours representing same degree of roughness are conventionally assigned fractographic terms, such as “mirror” or “mist” boundary; this subjective approach to qualitatively describe the fracture surface was considered necessary, due to the absence of an exact mathematical solution. We find that coordinate system transformations, applied to a dynamically adjusted classical fracture mechanics framework, enable these boundaries to be mathematically described by conic sections. A numerical dynamic crack evolution model was used to empirically validate this exact geometric description for model isotropic, monolithic materials, including borosilicate glass and vitreous or glassy carbon fractured in bending. As no arbitrary fractographic features are assigned, we can objectively quantify the applied stress at failure, the residual stresses originally present in the solid, and the mechanical material properties.

1. Introduction

Some of the early observations on fracture patterns can be attributed to the work of Broadmann [1] dating back to 1894. Broadmann [1] was particularly interested in the fracture of glass rods tested in tension, bending, and torsion. Broadmann [1] noted that fractographic features correlated to the strength of the glass, although a formal relationship correlating the strength of the material to the mirror radius was formally put forward only over half a century later by Orr [2]. Orr’s work on the fracture behavior of glass samples gave an approximate description of the shape of the mirror-mist boundary for samples fractured in tension, but failed to quantitatively describe the shape of the mirror-mist boundary for flexural fractures. During the last half-century, Orr’s equation has become the standard for fractographic analysis [3–7].

Shand [8] corrected the shape of the mirror-mist boundary by including the non-uniform stress concentration factor around the crack tip. Shand [8] assumed that the mirror-mist boundary formed at a critical stress value; however, he only considered circular cracks in cylindrical glass rods. Johnson and Holloway [9] attempted to better predict the shape of mirror-mist boundary by using the empirical relationship $\sigma = A/\sqrt{R}$ and solving for the distance from the fracture origin, R, for a given stress field. Although this approach lacked physical foundation, it yielded reasonably accurate results for short cracks subjected to either flexural or tensile stress.

Kirchner and Kirchner [10] and Kirchner and Conway [11] improved upon the model proposed by Johnson and Holloway [9] by recognizing that the mirror-mist boundary, the hackle boundary, and the branching boundary, corresponded to

* Corresponding author.
E-mail addresses: rdugnani@alumni.stanford.edu (R. Dugnani), ricardo.zednik@etsmtl.ca (R.J. Zednik).

http://dx.doi.org/10.1016/j.engfracmech.2016.08.013

Nomenclature

a	crack length along the minor semi-axis
c	crack length along its major semi-axis
H	sample's thickness
K_I	stress intensity factor
R_b	branching radius
R_o	outer mirror radius
R_m	the 'mirror-mist boundary' radius
λ	ratio between the length of the crack's minor axis and the sample thickness
σ_b	bending stress
σ_m	membrane stress

distinct values of the stress intensity factor (SIF). Although Kirchner and Kirchner's solution [10] accounts for the static SIF calculated along the crack front, it only considers semi-circular cracks. This means that the SIF locus predicted by this method becomes less accurate for non-uniaxial stress states, such as samples fractured in flexion or bending, where the crack elongates in the direction of decreasing stress.

Tsai and Mecholsky [12] used an approach analogous to the one describe by Kirchner and Conway [11] to predict the formation of micro- and macro-branching in both isotropic and anisotropic brittle materials. In addition to the static stress intensity criterion, Tsai and Mecholsky proposed two alternative criteria to predict micro- and macro-branching boundaries, one based on constant strain intensity and the second on constant fracture energy. Qualitative comparison between the predicted shapes of the micro- and the macro-branching loci and the experimental data on silicon crystals suggested that, for anisotropic materials, the branching boundaries formation might be better predicted by the constant energy criterion. Nonetheless, for isotropic brittle materials, both the constant energy criterion and the constant stress intensity criterion are identical to the solution proposed by Kirchner and Conway [11].

Shetty et al. [13] expand upon Kirchner and Kirchner's approach [10] by allowing other than circular cracks. Shetty's implementation is based on semi-static and fatigue data, and is therefore limited by the assumption that the crack's aspect ratio varies as $a/c = 1 - a/t$. Since a/c changes at a significantly lower rate for dynamic cracks, Shetty's model [13] is unable to accurately predict the shape of the SIF locus. The effect of the stress field on the shape of specific SIF loci, such as the SIF locus corresponding to the mirror-mist boundary, was qualitatively described by the work of Quinn [14] and is now routinely used in the study of fracture behavior in brittle solids. Quinn [14] acknowledges that the mirror shapes are affected by stress gradients and that the shape is elongated in the direction of decreasing stress, but does not formulate a quantitative description.

Recently Plouraboué has shown that the fracture surface roughness is not simply the result of perturbation at the scale of the material microstructure: rather, the fracture surface roughness encompasses a broad spectrum of length scales, ranging from the atomic level to scales in the same order of magnitude of the fractured sample, with the latter contributing dominantly to the fracture roughness appearance [15]. Spectral analyses and other quantitative techniques have been used in the past to characterize the fracture's surface roughness and help characterize the mirror-mist and the mist-hackle boundaries. These techniques indicate [16,17] that the topography of the fracture surface displays a scale-invariant character, known as "self-affinity" or "self-similarity" [18]. Mandelbrot [19] and, more recently, the work of Balarkin [20] and Bouchbinder [21] have also shown that the fracture surfaces of many solids exhibit fractal or self-affine characteristics.

Wnuk and Yavari [22] proposed using an equivalent smooth blunt crack to describe the behavior of fractal and self-similar cracks. Although partially relying on dimensional analysis, the model found that the fracture roughness linearly increased as a function of the crack length, hence demonstrating the progressive formation of the mirror–mist–hackle regions. In this particular framework, the fracture roughness was shown to be a function of the SIF, the fractal exponent (or Hurst exponent), and the ratio of the crack tip radius to a characteristic length-scale of the material.

Xie and Sanderson [23,24] proposed an alternative method to describe the propagation of fractal cracks. Similarly to Wnuk and Yavari [22], in Xie and Sanderson's model, the dynamic SIF was also found to be a function of the fractal exponent and of a characteristic length-scale of the material.

Buehler and Gao's work [25] presented results from a large-scale molecular dynamics simulation describing the behavior of a two-dimensional, rough crack in brittle isotropic material. The molecular dynamics simulation showed that, as the speed of the crack increased, dynamical instabilities led to an increased roughening of the fracture surface. When the crack moved at low speeds, flat, mirror-like surfaces were formed. As the speed of the crack increased, the fracture surface turned rougher (i.e., 'mist'), and eventually became a very rough, irregularly faceted fracture surface (i.e., 'hackles'). A somewhat surprising, yet very important, conclusion drawn by Buehler and Gao's molecular dynamics simulations was that the stress distribution at the crack tip was consistent with the results from classic fracture mechanics up until crack-branching, hence justifying the use of classic fracture mechanics during the initial fracture surface development.

The recent work of Dugnani and Zednik [26] describes the angle of SIF loci relative to the free surface and numerically estimates the effect of the stress field on such an angle. This angle can therefore be used to estimate the strength of the sample. This approach was shown to apply to any isotropic brittle solid, including glass, fine grained ceramics or metals, and high