Engineering Fracture Mechanics 165 (2016) 153–182

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Crack initiation: A non-local energy approach

Costy Kodsi

School of Engineering, University of Glasgow, G12 8QQ, UK

ARTICLE INFO

Article history:
Received 11 March 2016
Accepted 6 June 2016
Available online 8 June 2016

Keywords:
Brittle fracture
Fracture criteria
LEFM
Topological derivative
PMMA

ABSTRACT

A crack initiation criterion is proposed for brittle material where nucleation is treated as a sudden and discrete rupture event at the macroscopic level. At the heart of the criterion is the finite difference form of the energy release rate; an expression for the characteristic length is derived and the change in total potential energy is obtained from an asymptotic argument. The criterion can predict crack onset at a sharp or blunt notch. Fracture toughness and material strength are the only input requirements. The effectiveness of the criterion is demonstrated through predictions that are compared to experimental data.

1. Introduction

Sokołowski and Żochowski in [1] introduced the topological derivative to provide the sensitivity of an arbitrary shape functional subject to an infinitesimal domain perturbation. This concept was duly applied by Van Goethem and Novotny in their study of crack nucleation [2]. Allaire et al. in [3] developed a damage model where the notion of the topological derivative was used to determine whether to nucleate damage in a healthy domain. The resulting criterion possesses attractive features, namely the simplicity of the analytical expression and ease of implementation in a numerical framework. However, the criterion is not appropriate in fracture mechanics.

This work is concerned with the development of a crack initiation criterion applicable to isotropic linear elastic media that is in a brittle state containing sharp and blunt notches, which yields the usual information: (i) fracture load; (ii) crack location; and (iii) propagation direction. Crack nucleation in this criterion is modelled as a sudden and discrete rupture event, i.e. a finite-length crack appearing abruptly, at the macroscopic level. The treatment of crack propagation as a discrete process is attributed to Novozhilov [4]. In the formulation of the criterion, the topological derivative is employed in the approximation of the total potential energy change associated with domain perturbation.

Fracture in this work is to be understood as the total separation of an initially intact body. The criterion is designed to operate within the bounds of Linear Elastic Fracture Mechanics (LEFM). Consequently, the stress field surrounding a notch tip is governed by linear elasticity theory and local nonlinear or dissipative behaviour will be considered negligible.

The paper is organised as follows. Section 2 provides the reader with the requisite mathematics and fracture mechanics knowledge. In Section 3, the topological derivative pertaining to the total potential energy functional is derived from first principles. This is followed with a review of common fracture criteria in Section 4. Section 5 presents the theoretical underpinnings of the proposed criterion. While in Section 6, predictions made by the criterion are scrutinised. Finally, concluding remarks are offered in Section 7.

E-mail address: c.kodsi.1@research.gla.ac.uk

http://dx.doi.org/10.1016/j.engfracmech.2016.06.006
0013-7944/

Nomenclature

a	variable crack length
a_c	characteristic length
$\mathcal{C}$	fourth-order isotropic elasticity tensor
da	differential crack extension
ds	differential surface element
dy	differential area element
D_T	first-order topological derivative
E	Young's modulus
F, F_c	load, subscript denotes critical value
$\mathbf{g}$	traction prescribed on the boundary
G, G_c	energy release rate, subscript denotes critical value
$\mathbf{h}$	displacement prescribed on the boundary
H_I, H_{II}	constants, Mode-I and Mode-II
$\mathbf{I}$	second-order identity tensor
$\mathbf{II}$	fourth-order identity tensor
J_I^{-1}, J_{II}^{-1}	coefficients, Mode-I and Mode-II
K_I	Mode-I stress intensity factor associated with a crack
K_I^η, K_{II}^η	generalised stress intensity factors, Mode-I and Mode-II
K_{Ic}	fracture toughness
l	existing crack length in body
m_{ij}^I, m_{ij}^{II}	angular functions, Mode-I and Mode-II
M_σ	stress failure function
$\mathbf{n}$	outward unit normal vector to boundary
N, N_c	normal force, subscript denotes critical value
p	finite length
P, P_c	tensile load, subscript denotes Mode-I fracture value
q	horizontal distance from centre of U-notched specimen
Q	change in total potential energy (domain perturbation)
$\mathcal{R}$	remainder function (asymptotic expansion)
S, S_c	strain energy density factor, subscript denotes critical value
$\mathbf{t}$	unit tangential vector to boundary
T	shear load
$\mathbf{u}$	displacement
$\mathbf{v}$	shape change velocity
V	shape change speed
w_c	modified McClintock criterion radial distance parameter
W	strain energy
$\mathbf{x}$	position vector relative to rectangular Cartesian axes $Ox_1x_2x_3$
$\mathbf{x}^\tau$	position vector relative to rectangular Cartesian axes $Ox_1^\tau x_2^\tau x_3^\tau$
Y	non-dimensional constant related to crack location
z_c	Novozhilov–Seweryn criterion characteristic length parameter
α	angle; $\alpha = \pi - \beta$
β	wedge semi-angle
χ	arbitrary shape functional
$\boldsymbol{\varepsilon}$	infinitesimal strain tensor
η_I, η_{II}	exponents, Mode-I and Mode-II
γ	fracture surface energy density
ι_c	shear strength
ς	constant representing plane stress or plane strain
λ	Lamé modulus
μ	shear modulus
ν	Poisson's ratio
ϕ_1, ϕ_2	coefficients; $\phi_1 = \frac{1}{2}(\sigma_1 + \sigma_2)$ and $\phi_2 = \frac{1}{2}(\sigma_1 - \sigma_2)$
φ	mapping (motion) function
Φ	scalar variable
Π	total potential energy functional
ψ	ratio of tensile to shear load
Ψ	scalar variable
ρ	notch radius