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Short cracks and V-notches: Finite Fracture Mechanics vs. Cohesive Crack Model

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ABSTRACT

In recent years, Finite Fracture Mechanics has proven to be an effective tool to estimate the strength of mechanical components, allowing fast strength predictions suitable for preliminary sizing and optimization of structures. In the present paper, we intend to corroborate the Finite Fracture Mechanics approach by showing that failure load estimates are very close to the ones provided by the well-established Cohesive Crack Model. To this aim, we consider two classical fracture mechanics problems, i.e. short cracks and V-notches. In the latter case, we believe to be of relevance also the Cohesive Crack Model semi-analytical solution herein provided.

1. Introduction

The Cohesive Crack Model (CCM) allows one to get accurate and physically-based strength predictions in plain or composite structural elements with stress concentrations or stress intensifications. Unfortunately, CCM usually requires a numerical implementation with large computing times that are not acceptable for preliminary sizing of structural details.

On the other hand, fast strength predictions can be obtained by applying the point stress criterion (or the average stress criterion). These methods predict failure when the stress at (or over) a certain distance (the so-called critical distance) reaches the material tensile strength. Nevertheless these approaches do not possess a clear physical background and show some drawbacks [1]; moreover, they require expensive experimental programs to identify the critical distances for different materials and geometries [2]. On the other hand, the recently introduced Finite Fracture Mechanics (FFM) allows one to overcome this shortcoming since the length of the critical distance is an outcome of the structural problem [1,3,4]. Furthermore FFM possesses a clear physical interpretation, i.e. fracture is supposed to propagate by finite steps. Thus, in the authors' opinion, FFM can be seen as the right candidate criterion to achieve accurate, physically-based and fast strength predictions.

Aim of the present paper is to corroborate this choice by showing that, for a couple of simple, yet relevant, case studies, the CCM and FFM strength predictions are in a very good agreement with each other. The two geometries to be investigated are represented by an infinite slab containing (i) a short crack and (ii) a (deep) re-entrant corner, both under simple mode I loading conditions. As well known, in both cases the Linear Elastic Fracture Mechanics (LEFM) fails in predicting the failure load. On the other hand, we will see that CCM and FFM correctly describe the transition from a toughness-governed failure to a strength-governed one, as the crack length decreases in the former case, and as the notch opening angle increases in the latter case. Noteworthy, both problems are solved in an almost completely analytical fashion.

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Nomenclature

(x,y)	spatial coordinates
F	concentrated force
σ_y	normal stress in y direction
σ_c	material tensile strength
σ	remote uniaxial stress
σ_f	remote failure stress
E'	Young modulus in plane strain conditions
$\mathcal{G}$	strain energy release rate
$\mathcal{G}_c$	fracture energy
a	crack length
a_p	process zone size
a_{pc}	process zone size at incipient failure
l_{ch}	Irwin length
Δ	crack increment
Δ_c	finite crack advancement
w	crack opening displacement
w_c	critical displacement
K_I	Mode I stress intensity factor
K_{Ic}	fracture toughness
K_I^*	generalized stress intensity factor
K_{Ic}^*	generalized fracture toughness
ω	notch opening angle
α	$\pi - \omega/2$, angle
λ	William's eigenvalue
$\gamma(\omega)$, $\beta(\omega)$, $\mu(\omega)$	shape functions
ξ	dimensionless generalized fracture toughness

Before starting to investigate the two geometries, it is worth observing that the agreement between CCM and FFM is to be expected, despite the different – continuous vs. discrete – crack growth mechanism, because they are both based on the same energy balance. The energy spent to create the new (unit) fracture surface is in fact $\mathcal{G}_c$ for both models (whereas the theory of critical distances usually does not fulfill this energy balance). A similar analogy between CCM and FFM holds also for the stress requirement: as well as the choice of the cohesive law is free for CCM, analogously the stress requirement to be coupled with the energy balance in FFM can be chosen arbitrarily (i.e. according to the material at hand). Moreover, once we fix the fracture energy and the tensile strength, the effect of the cohesive law shape as well as of the stress requirement expression is relatively weak for process zones/crack extensions much smaller than other geometrical lengths (see e.g. [5] for what concerns CCM).

Wishing to compare CCM and FFM, we expect similar predictions by CCM with a constant cohesive law and by the FFM approach with a point-wise stress requirement – as proposed by Leguillon [3]. Analogously, similar predictions are argued for CCM with a linearly descending cohesive law and for the FFM approach with an average stress condition – as proposed by Cornetti et al. [1] and applied to V-notches in [6]. In fact, the former choices provides a smaller process zone/crack extension but with a higher stress field, whereas the latter features yield a larger process zone/crack extension but with a lower stress field. This conjecture is confirmed for the pull–push shear test [7].

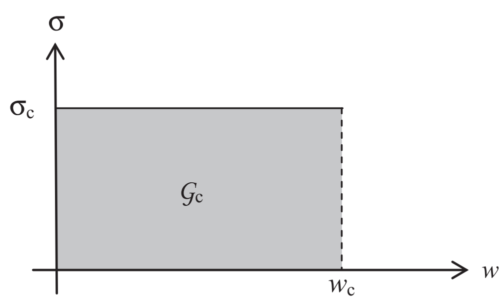

Fig. 1. Dugdale cohesive law.