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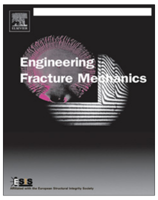

An 2-D SGBEM formulation of contact models coupling the interface damage and Coulomb friction in fibre–matrix composites

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ABSTRACT

A new numerical contact model able to predict the interface debonding and damage considering Coulomb friction contact between debonded surfaces has been developed. A comparison of two variants of this model is presented from the mathematical point of view, the Linear Elastic–Brittle Interface Model (LEBIM) and the Cohesive Interface Model (CIM), both combined with frictional contact. The paper discusses the process of the interface debonding in the two subsequent fracture steps: (i) the crack initiation and (ii) the crack propagation in the fibre–matrix interface. The concept of the solution is approximated by a time-stepping procedure and a boundary element approach.

1. Introduction

Recently, the advance in development and analysis of composite laminate materials considerably influenced the applicability of fibre-reinforced composites in the area of aeronautical, aircraft and civil engineering industry. The scientific research interested in developing of applications of the fibre–matrix delamination is increasing, above all the applications in aerospace industry where the combination of lightweight and structural responsibility is a key aspect of design. Therefore, the analysis of relevant theoretical approaches and developing of mathematical models for investigation of failure mechanisms in fibre-reinforced composites seems to be crucial for applicability in engineering practise. One of the most common damage mechanisms occurring in composite unidirectional laminates on the micro-, meso- and macro-scale levels is associated to the *matrix failure* well known as *inter-fibre failure*, in particular, when the laminate structure is subjected to tension dominated transversal load. Such applied tension loads are determining factors for the process of evolution of the fibre–matrix delamination.

The failure mechanism follows a well described sequence of damage steps: (i) crack initiation at the fibre–matrix interface by initial debonds of some fibres, (ii) the crack onset and growth of the debonds at the interface and then (iii) kinking out of the interface crack across the matrix and (iv) mergence of the growing fibre–matrix interface cracks resulting in nucleation of a macro crack which may cause the overall failure of the unidirectional lamina. This non-linear problem of an elastic circular (2D) or cylindrical (3D) fibre inclusion embedded in an infinite elastic matrix subjected to remote uniaxial load has been intensively analysed by many authors, see [7,9–11,27,29,34,35,46,47].

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Nomenclature

$\mathbf{C}$	the fourth-order stiffness tensor [MPa]
$\mathbf{D}$	the fourth-order tensor of viscosity [MPa s]
ε	the stored energy functional [J]
E_m	Young's modulus of matrix [MPa]
E_f	Young's modulus of fibre [MPa]
$\mathcal{F}$	the potential energy of the external forces [J]
f	the external forces [MPa]
$\mathbf{f}$	an increment of the external loading [MPa s^{-1}]
c	the power coefficient (RCCM model) [–]
a_0	the fracture energy of the debonded adhesive (RCCM model) [J m^{-2}]
f_2^k	the incrementally prescribed vertical loading [MPa s^{-1}]
G	the driving force [J m^{-2}]
G_d	the fracture energy [J m^{-2}]
k	the load step [–]
k_n	the interface tangential stiffness [MPa μm^{-1}]
k_s	the interface normal stiffness [MPa μm^{-1}]
k_g	the normal stiffness parameter of compressibility [MPa μm^{-1}]
L	the dimension of the matrix [μm]
p	the friction function power coefficient [–]
q	the friction function parameter [–]
r	the radius of a fibre inclusion [μm]
$\mathcal{R}$	the dissipation potential [J s^{-1}]
$\mathbf{t}$	the traction vector [MPa]
t_s	the tangential component of the traction vector [MPa]
t_n	the normal component of the traction vector [MPa]
t_{n_c}	the critical normal stress [MPa]
t_{s_c}	the critical tangential stress [MPa]
$\mathbf{u}$	the displacement vector [μm]
u_n	the normal component of the displacement vector [μm]
u_s	the tangential component of the displacement vector [μm]
u_{n_c}	the critical normal displacement [μm]
u_{s_c}	the critical tangential displacement [μm]
u_s^s	the plastic (slip) part of the tangential displacement [μm]
u_s^e	the elastic (stick) part of the tangential displacement [μm]
$[\dot{\mathbf{u}}]_s$	the relative sliding velocity [μm s^{-1}]
$[\mathbf{u}]_n$	the relative normal displacement [μm]
$[\mathbf{u}]_s$	the relative tangential displacement [μm]
η	the symbol of the pertinent domain (boundary) [–]
Ω^η	the domain [–]
Γ^η	Lipschitz boundary [–]
Γ_c	the interface [–]
ω	the interface viscosity parameter [s]
κ	the number of inclusions [–]
ν_m	Poisson's ratio of matrix [–]
ν_f	Poisson's ratio of fibre [–]
ζ	the damage parameter [–]
$f(\zeta)$	the damage dependent friction function [–]
$\phi(\zeta)$	the damage dependent stiffness function [–]
τ_R	the bulk viscosity parameter [s]
τ_0	the initial increment of time step [s]
τ^k	the *k*th increment of time step [s]

1.1. Energetic approach

The solved non-linear problem includes various phenomena which provide highly complex multi-domain and contact layouts. Nowadays, there exist several theoretical approaches how to describe the process of delamination. Presented model follows energy-based principles, where the philosophy of quasistatic evolution of crack propagation process at the interface

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