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Crack nucleation in negative geometries ☆

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ABSTRACT

The failure behaviour of structural situations depends on the shape of the energy release rate function of possible crack configurations. Depending on the structural situation, the energy release rate function can decrease (negative geometry) or increase (positive geometry) with increasing crack size. In this work, three structural situations exhibiting locally positive and locally negative behaviour are analysed using a coupled stress and energy criterion in the framework of Finite Fracture Mechanics. The nucleation and stability of finite cracks is discussed in detail. Exact and approximate solutions for the stress fields and the stress intensity factors are employed or newly derived to allow for a closed-form solution of the failure model. For the considered configurations, it is identified whether the nucleated cracks are stable or unstable.

1. Introduction

It has been shown that Finite Fracture Mechanics (FFM) can be used efficiently to analyse crack nucleation at stress concentrations. Analyses of crack nucleation at weak stress singularities as they arise at V-notches [15,19,49], due to the laminate free edge effect [24,37] or in bi-material joints [33,46,52] have been performed successfully. Also stress concentrations without stress singularities can be studied advantageously using FFM, and the results correctly cover occurring size effects, e.g. in plates with open holes [2,9,53] or U-notches [11,43]. A major feature of FFM solutions is that no characteristic length scale is required for the failure evaluation but a physically sound failure model is employed. This constitutes an advantage over approaches following the Theory of Critical Distances or Inherent Flaw Models [48] that use an empiric length parameter which has to be identified by specific experiments for each structural situation and loading. In FFM, the fundamental material parameters strength and fracture toughness suffice for failure evaluation.

In Fracture Mechanics, structural situations can be classified into negative and positive geometries [4,5,26]. In negative geometries, cracks are stable after onset, while in positive geometries, cracks are unstable after onset. Stable cracks are associated with an energy release rate that is a decreasing function of the crack length and, hence, increasing loads are required for crack growth.

This classification is used in Linear Elastic Fracture Mechanics (LEFM) and relates to the growth of cracks. In Finite Fracture Mechanics, the formation of cracks is analysed and the further development of the crack can be studied by means of LEFM, as long as the crack does not approach interfaces [34,35] or alike. The overall behaviour of the energy release rate not only affects the crack growth of the initiated cracks but also has a major effect on the analysis of crack nucleation by FFM. For example, if the energy release rate depends on the length of the crack, the considered structural situation can have

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Nomenclature

a	crack semi length
f	equivalent stress
K	stress intensity factor
K_{Ic}	critical stress intensity factor
l_{ch}	Irwin's length
p	nodal forces
P	point load
R	hole radius
$u,\ v$	nodal displacements
$x,\ y$	coordinates
Δa	finite crack length
Π	total potential
σ_c	strength
σ_{ij}	stress
Ω_c	crack surface
$\bar{K}$	integrated stress intensity factor
$\bar{\mathcal{G}}$	incremental energy release rate
$\mathcal{G}$	energy release rate
$\mathcal{G}_c$	critical energy release rate

a range of crack configurations for which it shows the behaviour of a negative geometry (locally negative) but for other crack configurations the behaviour of a positive geometry prevails (globally positive).

In this work, crack nucleation is assessed by means of FFM for three structural situations: The point-loaded centre crack showing a globally negative behaviour (Section 2.2), the point-loaded open hole showing a globally negative but locally positive behaviour (Section 2.3) and the single-Lap-Joint with globally positive but locally negative behaviour (Section 2.4). The FFM solutions and their prerequisites are given in detail and the solutions are discussed particularly with regard to the general features of FFM solutions in these cases.

2. FFM analyses of negative geometries

2.1. Prerequisites

The formation of cracks can be understood as governed by two necessary conditions: a condition of sufficiently large local loading of the structure and a second condition of energetic permissibility. Using the concept of Finite Fracture Mechanics (FFM) [23], which considers the instantaneous formation of cracks of finite size, Leguillon [32] has proposed a coupled stress and energy criterion. Considering the formation of a crack of finite size Δa with the crack surface denoted Ω_c, the coupled criterion can be written as

$$f(\sigma_{ij}(\vec{x})) \geqslant \sigma_c\, \forall \vec{x} \in \Omega_c(\Delta a) \quad \wedge \quad -\frac{\Delta\Pi(\Delta a)}{\Delta a} \geqslant \mathcal{G}_c \tag{1}$$

Here, f is an appropriately chosen stress function (e.g. the maximum principal stress) and $\Delta\Pi(\Delta a)$ is the change of the potential energy due to the formation of the finite crack. Two failure parameters, strength σ_c and the critical energy release rate $\mathcal{G}_c$, which is a measure of fracture toughness [1], are required.

The introduced quantity of an energy release rate of a finite crack is called incremental energy release rate and is denoted with $\bar{\mathcal{G}}$:

$$\bar{\mathcal{G}} = -\frac{\Delta\Pi(\Delta a)}{\Delta a}. \tag{2}$$

It is the average of the differential energy release rate and, thus, generally different from it:

$$\bar{\mathcal{G}} = \frac{1}{\Delta a}\int_0^{\Delta a} \mathcal{G}(\tilde{a})\mathrm{d}\tilde{a} \quad \text{and equally} \quad \frac{\mathrm{d}(\bar{\mathcal{G}}\cdot\Delta a)}{\mathrm{d}\tilde{a}} = \frac{\mathrm{d}\bar{\mathcal{G}}}{\mathrm{d}\tilde{a}}\Delta a + \bar{\mathcal{G}} = \mathcal{G}(\tilde{a}). \tag{3}$$

As can easily be seen, the differential energy release rate is smaller than the incremental one when its derivative with respect to the crack length is negative $\partial\bar{\mathcal{G}}/\partial a < 0$ and vice versa. Consequently, intersections occur at extrema of the incremental energy release rate ($\partial\bar{\mathcal{G}}/\partial a = 0$).