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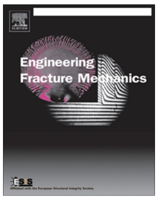

Approximation of mixed mode propagation for an internally pressurized circular crack

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ABSTRACT

Hydraulic fracturing of rocks has various engineering applications. However, there has been limited research into crack propagation prediction by three dimensional analytical techniques. This paper discusses such a technique for predicting the propagation surface of a pressurized circular crack subjected to various loading conditions. The propagation surfaces predicted from the proposed crack front propagation algorithm align well with published results. The suggested method consumes only a fraction of the time needed for a numerical simulation, and therefore it could be useful in assisting the design of hydraulic fracturing operations.

1. Introduction

It is important to predict the propagation paths of pressurized cracks in hydraulic fracturing operations in order to design and optimize the extraction of resources, such as geothermal energy and unconventional oil and gas. Hydraulic fracturing is the primary and most effective method used to increase productivity in these applications [1,2] as it can enhance the rock mass permeability significantly via the resultant fractures.

In an industrial setting, a method is needed for the quick initial assessment of the resultant fracture propagation surface according to the local stress conditions [3]. Establishing such an analytical method has clear advantages including ease of implementation and quick processing times. Numerical methods to address the problem exist [4], however, most of them have been limited to two dimensional [5,6] or highly intensive computational methods [7]. The numerical method proposed by Huang et al. [4] aims to solve a similar problem addressed by this paper. Their numerical method uses a virtual multidimensional internal bond model that is implemented in a three dimensional finite element code.

Rahman et al. [3] developed a two dimensional analytical method to predict the propagation of inclined cracks. Their calculated two dimensional stress intensity factors were close to those obtained from the boundary element analysis using FRANC3D. The propagation paths from their case studies exhibited close alignment with those obtained from FRANC3D and literature. To the knowledge of the authors, the work presented in this paper is the first attempt to extend the method described in Rahman et al. [3] to three dimensional applications.

The key advantage of the three dimensional methods is that they allow determination of the way a pre-existing crack re-orientates in the presence of various in-situ compressive stress regimes. In order to design effective hydraulic fracturing operations, it is important to determine this resultant crack propagation surface since the resultant fracture network

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Nomenclature

a	radius or major axis of the elliptical crack (m)
a_{median}	median crack increment input for FRANC3D (m)
b	minor axis of the elliptical crack (m)
inc	predefined incremental length for the proposed analytical method (m)
$inc_{FRANC3D}(\varphi)$	incremental length used in FRANC3D (m)
$K_{I(median)}$	median stress intensity factor for mode I along the crack front (Pa $\sqrt{\mathrm{m}}$)
$K_I(\varphi)$	stress intensity factor for mode I (Pa $\sqrt{\mathrm{m}}$)
$K_{II}(\varphi)$	stress intensity factor for mode II (Pa $\sqrt{\mathrm{m}}$)
$K_{III}(\varphi)$	stress intensity factor for mode III (Pa $\sqrt{\mathrm{m}}$)
$K_{I(kinked)}(\varphi)$	kinked crack analytical stress intensity factor for mode I (Pa $\sqrt{\mathrm{m}}$)
$K_{II(kinked)}(\varphi)$	kinked crack analytical stress intensity factor for mode II (Pa $\sqrt{\mathrm{m}}$)
$K_{III(kinked)}(\varphi)$	kinked crack analytical stress intensity factor for mode III (Pa $\sqrt{\mathrm{m}}$)
n	power input for calculating the incremental length in FRANC3D
α	dip direction (°)
β	dip angle (°)
γ	ellipse angle – the direction from the projected dip direction on the crack plane to the major axis of the ellipse (°)
θ	crack front angle – from the normal to the crack front towards the positive z axis direction (°)
$\theta_c(\varphi)$	critical crack front angle (°)
$\theta_{kink}(\varphi)$	difference from the radial vector of the current fictitious plane to the kinked radial line (°)
υ	Poisson's ratio
$\sigma_{n(eff)}$	effective normal stress on the surface of the crack (Pa)
$\sigma_{n(external)}$	normal stress on the surface of the crack (Pa)
σ_t	tensile strength (Pa)
τ	shear stress along the surface of the crack (Pa)
τ_{eff}	effective shear stress along the surface of the crack (Pa)
φ	crack front angle – from the x axis direction clockwise around the normal vector in the positive z axis direction (°)
ω	shear angle – clockwise around the normal vector in the positive z axis direction (°)
$B = (k^2 - \upsilon)E(k) + \upsilon k'^2 K(k)$	constant used to calculate the elliptical crack stress intensity factors
$C = (k^2 + \upsilon k'^2)E(k) - \upsilon k'^2 K(k)$	constant used to calculate the elliptical crack stress intensity factors
$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2(\alpha)}d\alpha$	elliptical integral of the second kind
$k = \sqrt{1 - \left(\frac{b}{a}\right)^2}$	intermediate eccentricity parameter used to calculate the elliptical crack stress intensity factors
$k' = \frac{b}{a}$	ratio of the minor to the major axis of the elliptical crack
$K(k) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1-k^2 \sin^2(\alpha)}}$	elliptical integral of the first kind
LEFM	linear elastic fracture mechanics

provides the major permeable pathways for fluid or gas flow. A properly established three dimensional hydraulic fracturing propagation model can assist in the design of the stimulated fracture network to better target the gas zone or geothermal energy resources. Therefore, it is of significant practical importance to develop an efficient and accurate method to predict the three dimensional propagation surfaces resulting from hydraulic fracturing.

To assist the development of such a mixed mode propagation method, the primary problem is simplified to a uniformly pressurized circular crack in an infinite medium subjected to uniform far-field stresses. Kassir and Sih [8] developed an analytical method to evaluate stress intensity factors of all three fracturing modes (opening, shearing and tearing) for a circular and an elliptical planar crack given an arbitrary loading regime based on the linear elastic fracture mechanics (LEFM) theory. It is well documented that these fracturing modes have a combined effect on the resultant crack propagation surface in rocks and other brittle materials [9–12].

In this paper, an analytical approach is developed to approximate the mixed mode propagation of a circular crack, by considering a fictitious equivalent elliptical crack and utilizing the maximum tangential stress criterion [13]. This fictitious elliptical crack assumes the surface and front formed by the propagation process are on the same plane. This assumption proved effective and satisfactory as the predicted propagation surfaces closely align with published results. Another simplification is using the effective normal and shear stresses to calculate the stress intensity factors, which is justified by the close alignment of their resultant values with the numerical results obtained from FRANC3D [14]. The stress tensor for the fictitious planar crack is also comparable to the stress tensor obtained from ABAQUS for the equivalent kinked crack. The finite element