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# Weight functions for a finite width plate with single or double radial cracks at a circular hole

Jihwi Kim, Michael R. Hill *

*Department of Mechanical and Aerospace Engineering, University of California, Davis, One Shields Avenue, Davis, CA 95616, United States*

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ABSTRACT

This work develops accurate weight functions for a single crack at a hole in a finite width plate for various hole sizes. In order to develop an accurate weight function, we first obtain accurate stress intensity factors, using the finite element method (FEM), for a reference load case of uniform stress on the crack line. Following the earlier approach for developing a weight function suggested by Wu and Carlsson, we fit the reference stress intensity factor data from FEM to a smooth analytic function; however, for the open hole it is necessary to adopt a piecewise polynomial to fit the stress intensity factor data, in place of the single polynomial suggested by Wu and Carlsson. We validate the new weight function for the case of remote uniform applied stress, which induces a stress field on the crack line exhibiting the well-known stress concentration at the hole, and for which we have accepted stress intensity factor solutions. The new weight functions provide stress intensity factors that agree very well with results from two commercial fracture mechanics software packages. Comparing results from the new and earlier weight functions shows good agreement for some crack line stress fields, but errors of a few percent for other stress fields, with the new weight function providing more reasonable results. The improved quality of the new weight functions is due both to the new reference solution for uniform crack line stress and to the piecewise fit to the reference stress intensity data. Trivial changes to the FEM model allow us to provide additional weight functions for the cases of symmetric double cracks at a hole (by adding a symmetry plane to the FEM mesh) and a single crack at a hole in a square plate (by reducing the length of the FEM mesh).


## 1. Introduction

Wu and Carlsson developed a library of weight functions for cracks in many different geometries [1]. Generally, these have good accuracy, but their weight function for a single crack at a hole in a long, finite width strip (Fig. 1) was found to have limited accuracy in recent work [2]. One inaccuracy arises because their solutions are for a limited range of geometry, and our earlier work used test coupons that fell outside of that range. But, there are additional causes of inaccuracy, as will be made clear below. Therefore, the goal in the present work is to develop a more accurate weight function for a crack at a hole in a finite width plate.

Weight functions are derived from a known reference load case (1), for which the stress intensity factor and crack face displacement are known [3,4]. The definition of the weight function is

* Corresponding author.
*E-mail address:* mrhill@ucdavis.edu (M.R. Hill).

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**Nomenclature**

| | |
|---|---|
| $a$ | crack size |
| $B$ | plate half width |
| $B_{j,d}$ | $j$th B-spline of order $d$ |
| $c_i$ | B-spline coefficients |
| $d$ | B-spline polynomial order |
| $E$ | Young's modulus |
| $E'$ | effective elastic modulus |
| $E_j(a/W)$ | auxiliary functions associated with the weight function of Wu and Carlsson |
| $F_i(a/W)$ | auxiliary functions associated with the weight function of Wu and Carlsson |
| $f_r(a/W)$ | geometry factor for a reference load case |
| $f(a/W)$ | geometry factor for an arbitrary load case |
| $G(a/W, x/a)$ | Green's function (stress intensity factor due to point loads on the crack face) |
| $g_i(a/W)$ | functions used to discuss contributions to the weight function |
| $I$ | order of polynomial fit to the reference stress intensity factor |
| $J$ | J-integral |
| $K(a)$ | stress intensity factor |
| $K_r(a/W)$ | stress intensity factor for a reference load case; reference stress intensity factor |
| $M$ | order of polynomial fit to the reference crack-line stress |
| $m$ | number of B-splines |
| $m(a, x)$ | weight function |
| $R$ | hole radius |
| $S_m$ | coefficients of the polynomial fit to the crack-line stress |
| $t_j$ | knot sequence for B-splines |
| $u(a, x)$ | vertical (opening) displacement of the crack face |
| $W$ | ligament width of a finite width plate with a hole ($W = B - R$) |
| $x$ | coordinate along the crack-line starting from the edge of a circular hole |
| $\alpha_i$ | coefficients of the polynomial fit to the reference stress intensity factor |
| $\beta_i(a/W)$ | factors in the weight function expression developed by Wu and Carlsson |
| $\phi(a/W)$ | auxiliary function associated with the weight function of Wu and Carlsson |
| $v$ | Poisson's ratio |
| $\sigma$ | stress magnitude used to normalize the stress field |
| $\sigma(x)$ | crack-line stress field in the uncracked body for an arbitrary load case |
| $\sigma_r(x)$ | crack-line stress field in the uncracked body for a reference load case |

$$m(a,x) = \frac{E'}{K^{(1)}(a)} \frac{\partial u^{(1)}(a,x)}{\partial a}, \tag{1}$$

where $a$ is the crack size, $x$ is the coordinate along the cracking-driving direction, with origin at the crack mouth, $E'$ is the effective elastic modulus ($E$ for plane stress and $E/(1-v)$ for plane strain), and $u^{(1)}(a,x)$ is the vertical (opening) displacement of the crack face under loading system (1). Given the weight function, the stress intensity factor $K^{(2)}(a)$ of any load system (2) may be found from the weight function $m(a,x)$ and the crack-line stress in the uncracked body due to load system (2), $\sigma^{(2)}(x)$. The specific expression for $K^{(2)}(a)$ is

$$K^{(2)}(a) = \int_0^a \sigma^{(2)}(x) \cdot m(a,x)dx. \tag{2}$$

Petroski and Achenbach [5] and Wu [6] showed that the crack face displacement of a center or edge crack can also be expressed as a function of the reference stress intensity factor, so the reference stress intensity factor is the only unknown and is the key factor related to the accuracy of a specific weight function. In addition, according to Wu and Carlsson [1], uniform stress on the crack-line is the best choice for the reference load case, because of its mathematical simplicity.

However, for cracks at a hole in a long strip [1], the only available reference solution is for remote uniform stress, for which accurate stress intensity factors are available [7]. In this case, the crack-line stress field is non-uniform, owing to the stress concentration at the hole. This allowed us to improve upon the earlier work of Wu and Carlsson. First, we developed a reference solution using uniform crack-line stress for a crack at a hole in a finite width strip. Second, piecewise polynomials were used to fit the reference stress intensity factor data, instead of using a single power series polynomial, as Wu had proposed [6]. This second step was required because the stress intensity factor solution for a crack at a hole has a rather complicated shape, with high gradients and curvature for both short and long cracks.

Following the same procedure as for a single crack in a long strip with an open hole, two additional weight functions were developed: (i) double-sided cracks in a long strip with an open hole, and (ii) a single crack in a square plate with an open